SUPERSYMMETRIC SYSTEMS WITH BENIGN GHOSTS

KIAS, Seoul, June 12, 2014

based on
A.S., J. Phys. A **47** (2014) 052001 [arXiv:1306.6066]
and several preceding papers

MOTIVATION:

Problems with causality in quantum and classical gravity.

Dream solution:

[A.S., 2005]

• our Universe as a soap film in a flat higher dimensional bulk. The TOE is a field theory in this bulk.

• To be renormalizable, it should involve higher derivatives

DANGER: the ghosts

• GHOSTS = instability (rather *absence*) of the vacuum

• inherent for higher-derivative theories.

Conventional system

$$E = \frac{\dot{q}^2}{2} + V(q)$$

can have a classical and/or quantum bottom

• Consider the Pais-Uhlenbeck oscillator.

$$L = \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 .$$

Then

$$E = \ddot{q}(\ddot{q} + \Omega^2 q) - \dot{q}(q^{(3)} + \Omega^2 \dot{q}) - \frac{1}{2}(\ddot{q} + \Omega^2 q)^2$$

can be as negative (and as positive) as one wishes.

• Common lore: negative residues in propagators break unitarity.

NOT TRUE!!

• No problem in free theories

• Interactions may lead to collapse and breaking of unitarity. Like falling into the center in the attractive potential $\sim 1/r^2$.

• If quantum theory is sick, so is its classical counterpart. If classical theory is benign, so is its quantum counterpart

The Ostrogradsky Hamiltonian of the free Pais-Uhlenbeck oscillator with inequal frequencies

$$L = \frac{1}{2}(\ddot{q} + \Omega_1^2 q)(\ddot{q} + \Omega_2^2 q) \ .$$

can be canonically transformed to

$$H = \frac{P_1^2 + \Omega_1^2 Q_1^2}{2} - \frac{P_2^2 + \Omega_2^2 Q_2^2}{2}$$

- The spectrum

$$E_{nm} = \hbar\Omega_1 \left(n + \frac{1}{2}\right) - \hbar\Omega_2 \left(m + \frac{1}{2}\right)$$

has neither top, nor bottom.

- pure point, dense everywhere.
- Classical motion is finite. Quantum problem is well defined. Evolution operator is unitary.

C. Bender + P. Mannheim, 2008 :

Let $\tilde{Q}_1 = Q_1$ and $\tilde{Q}_2 = iQ_2$.

• One can define then a spectral problem associated with the original PU spectral problem, which has the positive definite Hamiltonian,

$$\tilde{H} = \frac{\tilde{P}_1^2 + \Omega_1^2 \tilde{Q}_1^2}{2} + \frac{\tilde{P}_2^2 + \Omega_2^2 \tilde{Q}_2^2}{2}$$

• True, but not necessary. One can well manage without that...

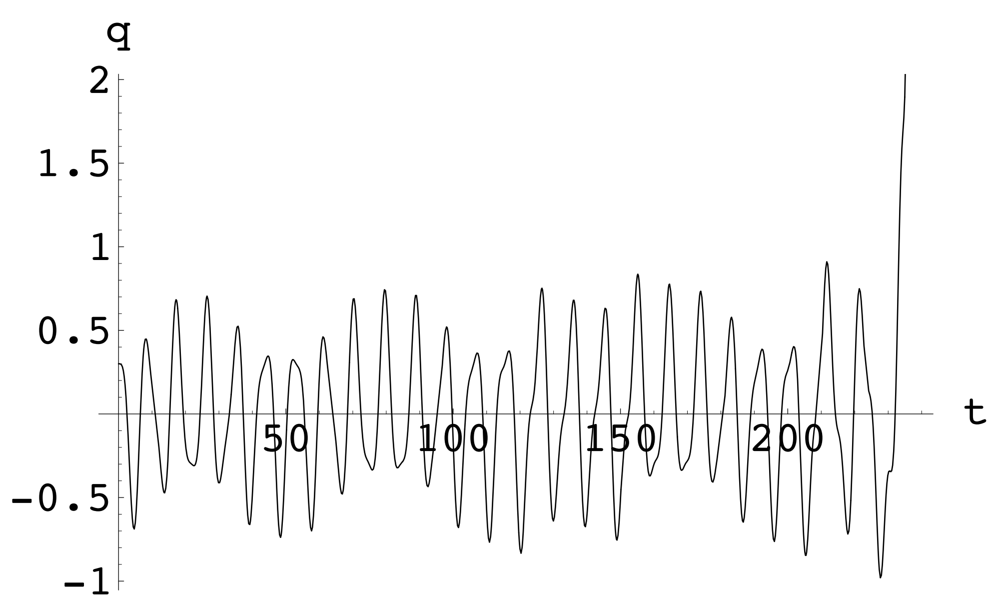
• Adding simple-minded nonlinear terms $\sim q^4$ or $\sim q^2\dot{q}^2$ to the Pais-Uhlenbeck Lagrangian does lead to collapse.

Islands of stability:

• Oscillates when $q(0)$ is small enough

• Runs to infinity at finite time when $q(0)$ is large enough.

The borderline case



Benign nonlinear SQM system with ghosts
[D. Robert + A.S., 2006]

$$S = \int dt dx d\bar{\theta} d\theta \left[\frac{i}{2} \bar{\mathcal{D}} \Phi \frac{d}{dt} \mathcal{D} \Phi + V(\Phi) \right] ,$$

with the real (0+1)-dimensional superfield

$$\Phi = \phi + \theta\bar{\psi} + \psi\bar{\theta} + D\theta\bar{\theta}$$

- An extra time derivative.

The Hamiltonian

$$H = pP - DV'(\phi) + \text{fermion term}$$

is not positive definite.

Integrals of motion:

1. $E \equiv H$
2. $N = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

- Exactly solvable.
- Take

$$V(\Phi) = -\frac{\omega^2\Phi^2}{2} - \frac{\lambda\Phi^4}{4}\,,$$

- Explicit solutions

$$\phi(t) \;=\; \phi_0 \, \mathrm{cn}[\Omega t|m]$$

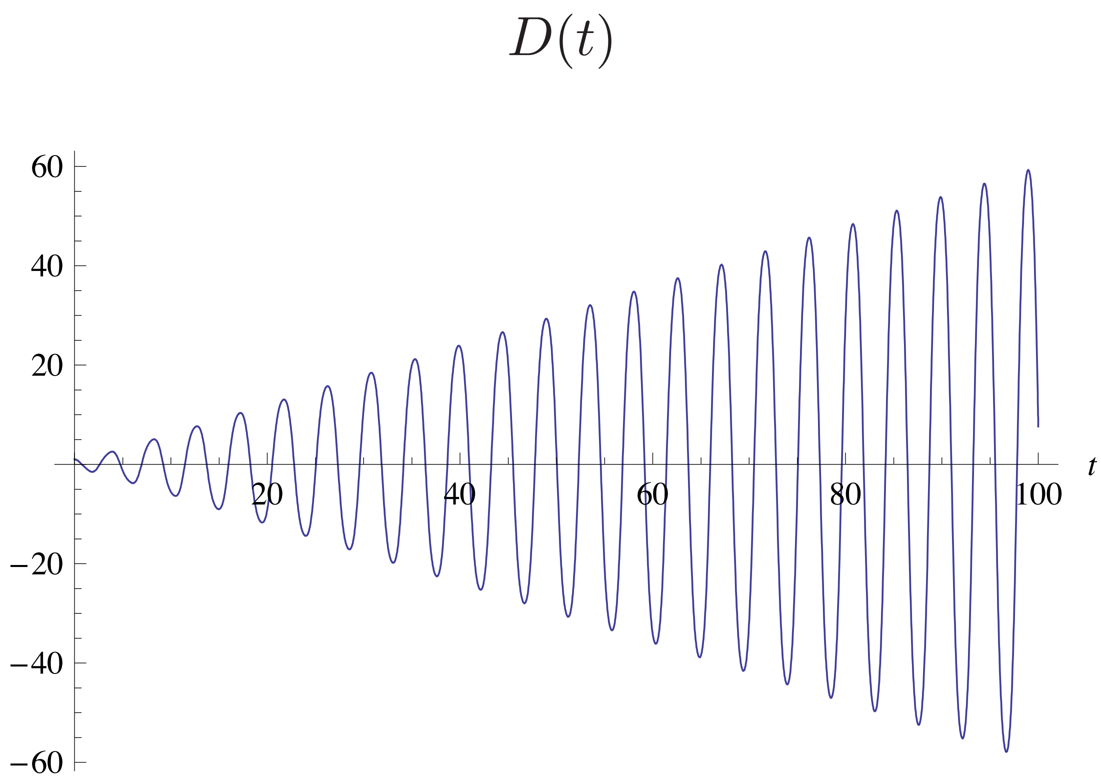
with

$$\alpha = \frac{\omega^4}{\lambda N}, \quad \Omega = [\lambda N(4+\alpha)]^{1/4}, \quad m = \frac{1}{2}\left[1 - \sqrt{\frac{\alpha}{4+\alpha}}\right],$$

$$\phi_0 \;=\; \left(\frac{N}{\lambda}\right)^{1/4} \sqrt{\sqrt{4+\alpha} - \sqrt{\alpha}}$$

$$D(t) \propto \dot{\phi}(t) \int^t \frac{dt'}{\dot{\phi}^2(t')}$$

- $\phi(t)$ is bounded.
- $D(t)$ grows linearly.



$D(t)$
60
40
20
−20
−40
−60
20
40
60
80
100
t

Quantum problem

- is also exactly solvable.
- If $\lambda = 0$, one can define $\{x_\pm, p_\pm\}$ such that

$$H_B = \frac{p_+^2 + \omega^2 x_+^2}{2} - \frac{p_-^2 + \omega^2 x_-^2}{2} \ .$$

with $E_{n_+,n_-} = \omega(n_+ - n_-)$.

- Infinite degeneracy at each level.

In interacting case:

- Still an infinity of zero energy states
- Other states form a continuous spectrum,

$E_{\text{cont}} \in [-\infty, -\omega] \cup [\omega, \infty]$.

Mixed model

$$L = \int d\bar{\theta} d\theta \left[\frac{i}{2} (\bar{\mathcal{D}}\Phi) \frac{d}{dt} (\mathcal{D}\Phi) + \frac{\gamma}{2} \bar{\mathcal{D}}\Phi \mathcal{D}\Phi + V(\Phi) \right] .$$

Physics is similar to the model with $\gamma = 0$, but

- not integrable anymore
- No linear growth for $D(t)$

.

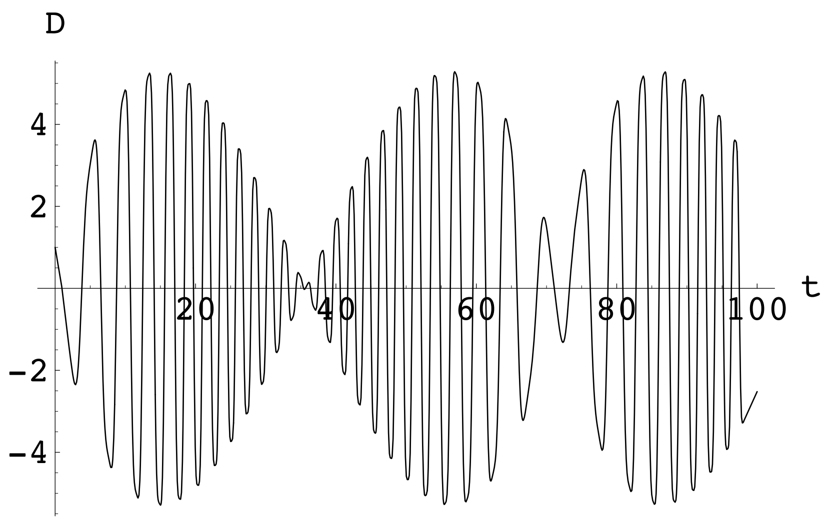


Figure 1: The function $D(t)$ for a deformed system ($\omega = 0, \lambda = 1, \gamma = .1$).

- Explicit expressions for wave functions exist.
- The evolution operator is unitary.

Spectrum

In noninteracting case ($\lambda = 0$),
$E_{n_+,n_-} = \omega_+ n_+ - \omega_- n_-$
with

$$\omega_\pm = \frac{\gamma}{2}\left(\sqrt{1+\tau^2} \mp 1\right) \qquad , \tau = \frac{2\omega}{\gamma}.$$

• The same as for the Pais-Uhlenbeck oscillator with different frequencies,

$$L = \frac{1}{2}\left(\ddot{q}^2 - (\omega_+^2 + \omega_-^2)\dot{q}^2 + \omega_+^2\omega_-^2 q^2\right)$$

• pure point, dense everywhere

• similarly - in the interacting case.

- Remark: the full canonical Hamiltonian

$$H = pP - DV'(x) - \frac{\gamma}{2}\left(D^2 + P^2\right)^2 + \bar{\psi}\bar{\chi} + \left(\frac{\gamma^2}{4} - V''(x)\right)\chi\psi\,.$$

is not Hermitian

- Its spectrum is, however, real. Belongs to the class of crypto-Hermitian Hamiltonians.

UNUSUAL ALGEBRAIC STRUCTURES

- canonical Nöther supercharges

$$Q = \psi[p + iV'(x)] - \left(\bar{\chi} + \frac{\gamma}{2}\psi\right)(P - iD) \ ,$$

$$\bar{Q} = \left(\bar{\psi} - \frac{\gamma}{2}\chi\right)(P + iD) - \chi[p - iV'(x)] \ .$$

- and the extra pair

$$T = \psi[p - iV'(x)] + \left(\bar{\chi} + \frac{\gamma}{2}\psi\right)(P + iD) \ ,$$

$$\bar{T} = \left(\bar{\psi} + \frac{\gamma}{2}\chi\right)(P - iD) + \chi[p + iV'(x)] \ .$$

• When $\gamma = 0$, we have a semidirect product of the standard $\mathcal{N} = 4$ SUSY algebra

$$\{Q, \bar{Q}\} = \{T, \bar{T}\} = 2H$$

(but $\bar{Q} \neq Q^\dagger, \ \bar{T} \neq T^\dagger$!)

and the Abelian Lie algebra generated by

$$N = \frac{P^2}{2} - V(x)\,, \qquad F = \psi\bar{\psi} - \chi\bar{\chi}$$

Nonvanishing commutators

$$\{Q, \bar{Q}\} = \{T, \bar{T}\} = 2H;$$
$$[\bar{Q}, F] = \bar{Q}, \ [Q, F] = -Q, \ [T, F] = -T, \ [\bar{T}, F] = \bar{T};$$
$$[Q, N] = [T, N] = \frac{Q - T}{2}, \quad -[\bar{Q}, N] = [\bar{T}, N] = \frac{\bar{Q} + \bar{T}}{2}\ .$$

- When $\gamma \neq 0$, the algebra is deformed
- Let $H = H_0 - \gamma F/2$ and introduce $F_+ = \bar{\chi}\psi, \quad F_- = \bar{\psi}\chi$

then

$$
\begin{aligned}
[F_\pm, F] = \mp 2F_\pm, \quad [F_+, F_-] = F \ , \\
[Q, H_0] = -\frac{\gamma}{2}Q, \ [\bar{Q}, H_0] = \frac{\gamma}{2}\bar{Q}, \\
[T, H_0] = \frac{\gamma}{2}T, \ [\bar{T}, H_0] = -\frac{\gamma}{2}\bar{T} \ , \\
[Q, F] = -Q, \quad [\bar{Q}, F] = \bar{Q}, \\
[T, F] = T, \quad [\bar{T}, F] = -\bar{T} \ , \\
[Q, F_-] = \bar{T}, \ [\bar{Q}, F_+] = -T, \\
[T, F_-] = -\bar{Q}, \quad [\bar{T}, F_+] = Q \ , \\
\{Q, \bar{Q}\} = 2H_0 - \gamma F, \quad \{T, \bar{T}\} = 2H_0 + \gamma F, \\
\{Q, T\} = 2\gamma F_+, \quad \{\bar{Q}, \bar{T}\} = 2\gamma F_- \ .
\end{aligned}
$$

- This is $osp(2,2)$ algebra.
- a close relative of weak supersymmetry algebra [A.S., PLB 585 (2004) 173]

(1+1) FIELD THEORY.

- Let Φ depend on t and x. Choose

$$S = \int dt dx d\bar{\theta} d\theta \left[-2i\mathcal{D}\Phi\partial_+\mathcal{D}\Phi + V(\Phi)\right] ,$$

where $\partial_\pm = (\partial_t \pm \partial_x)/2$ and

$$\mathcal{D} = \frac{\partial}{\partial\theta} + i\theta\partial_-, \qquad \bar{\mathcal{D}} = \frac{\partial}{\partial\bar{\theta}} - i\bar{\theta}\partial_+$$

Bosonic Lagrangian

$$\mathcal{L}_B = \partial_\mu\phi\partial_\mu D + DV'(\phi)$$

Equations of motion

$$\begin{aligned} \Box\phi + \omega^2\phi + \lambda\phi^3 &= 0 \\ \Box D + D(\omega^2 + 3\lambda\phi^2) &= 0 . \end{aligned}$$

Two integrals of motion:

$$E = \int dx \left[\dot{\phi}\dot{D} + \phi' D' + D\phi(\omega^2 + \lambda\phi^2) \right]$$

(positive or negative)

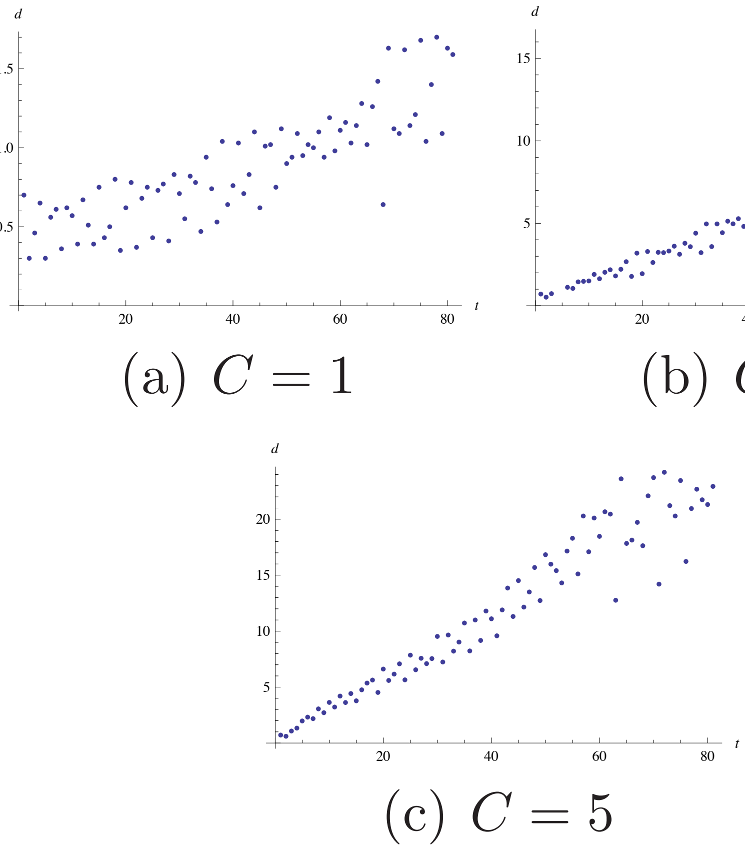
$$N = \int dx \left[\frac{1}{2}\left(\dot{\phi}^2 + (\phi')^2 \right) + \frac{\omega^2\phi^2}{2} + \frac{\lambda\phi^4}{4} \right]$$

(positive definite)

- Stochasticity. Solved numerically.

Initial conditions

$$\begin{aligned} \phi(x, t=0) &= Ce^{-x^2}, \quad \text{with} \quad C = 1, 3, 5 \\ D(x, t=0) &= \cos \pi x / L \\ (L = 10 &- \ \text{length of the box}) \end{aligned}$$



(a) $C = 1$

(b) $C = 3$

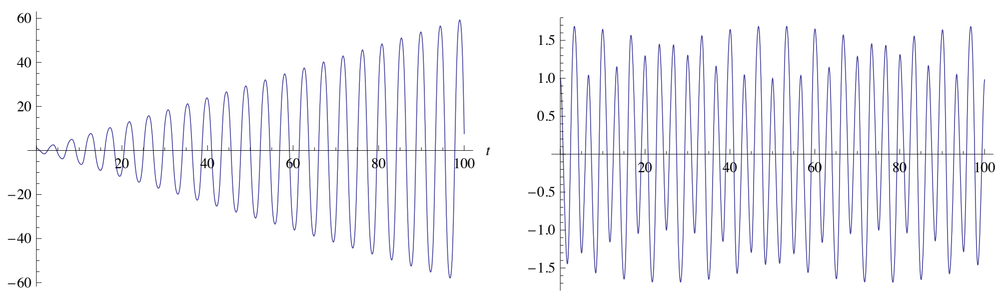
(c) $C = 5$

Figure 2: Dispersion $d = \sqrt{\langle D^2 \rangle_x}$ as a function of time for different values of C.

IMPLICATIONS FOR INFLATION ?

- Homogeneous classical field needed.
- Let $\phi(t)$ be homogeneous. Then different Fourier modes of $D(x,t)$ decouple.

ONLY THE ZERO MODE GROWS !



(a) $k=0$ (b) $k=1$

- $D(x,t)$ becomes more and more homogeneous.

TALK AFTER TALK:

a 6D superconformal theory

Matter content: bosons: non-Abelian gauge fields A_M $(M = 0, 1, \ldots, 5)$ and adjoint scalars $D_{jk} = D_{kj}$ $(j, k = 1, 2)$

fermions: adjoint 6D Weyl spinors $\psi^a_{\alpha j}$ $(\alpha = 1, 2, 3, 4)$

Component Lagrangian

$$
\begin{aligned}
S \;\; = \;\; & -\frac{1}{g^2}\int d^6x \,\mathrm{Tr}\Big\{\left(\nabla^M F_{ML}\right)^2 + i\psi^j\gamma^M\nabla_M(\nabla)^2\psi_j + \frac{1}{2}\left(\nabla_M\mathcal{D}_{jk}\right)^2 \\
& + \mathcal{D}_{lk}\mathcal{D}^{kj}\mathcal{D}_j{}^l - 2i\mathcal{D}_{jk}\left(\psi^j\gamma^M\nabla_M\psi^k - \nabla_M\psi^j\gamma^M\psi^k\right) + (\psi^j\gamma_M\psi_j)^2 \\
& + \frac{1}{2}\nabla_M\psi^j\gamma^M\sigma^{NS}[F_{NS}, \psi_j] - 2\nabla^M F_{MN}\,\psi^j\gamma^N\psi_j\Big\}
\end{aligned}
$$

• Dimensionless coupling

• Classical superconformal symmetry

• Broken at the quantum level. Asymptotic freedom

$$g(\mu) \;=\; g_0 \left(1 - \frac{5 g_0^2 c_V}{48\pi^3} \ln \frac{\Lambda_{UV}}{\mu} \right)$$

• Chiral anomaly. Can be cancelled by adding an adjoint hypermultiplet.

• Malignant ghosts and collapse. Especially clearly seen in the scalar sector.

Eq. mot. $\ddot{D} \sim D^2$. Reaches infinity at finite time.