Recent Development in Theoretical Physics

  

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Bubbles and Walls : Revisited

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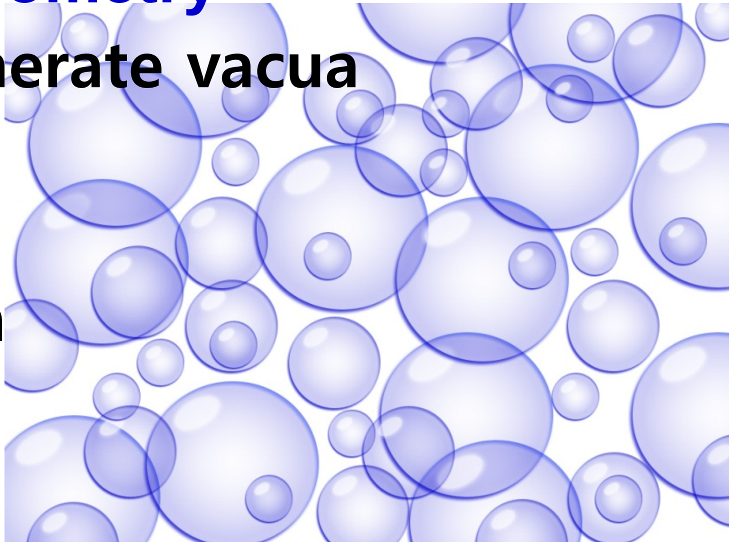
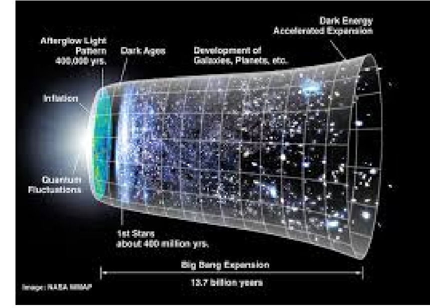


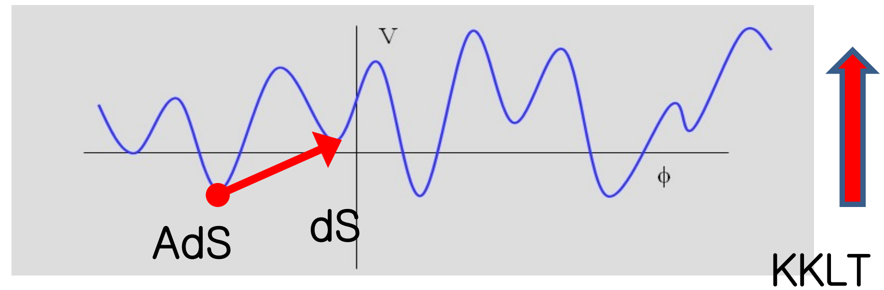
5. Summary and discussions

1. Motivation



◆ Observation : Universe is expanding with acceleration
– needs positive cosmological constant

◆ The string theory landscape provides a vast number of metastable vacua.



◆ How can we be in the vacuum with positive cosmol const ?

- KKLT(Kachru,Kallosh,Linde,Trivedi, PRD 2003)) : “lifting” the potential
- an alternative way? → Revisit Bubbles & Walls : Gravity effect important

◆ Bubbles – for Phase Tran. in the early universe w/ or w/o cosmol. const,
Can there be the nucleation of a false vac bubble? Can it expand?
Other kinds of Bubbles? (Including Gravity Effects)

(*) Other possible roles of the Euclidean Bubbles and Walls?
Ex) quantum gravity ? Cosmology such as domain Wall Universe?

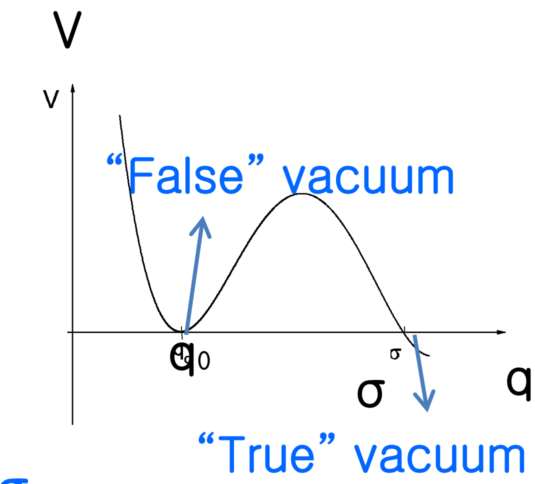
We will revisit and analyse vacuum bubbles in various setup.

2. Basics – Reivew

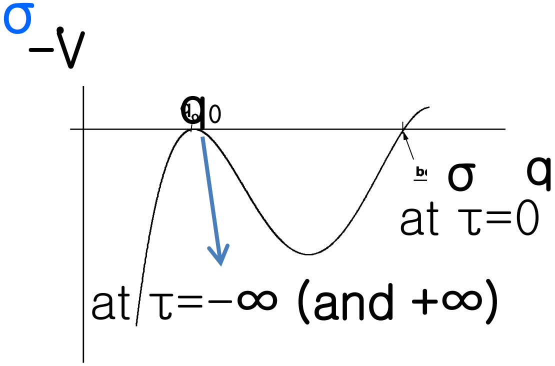
(1) Tunneling in Quantum Mechanics

- particle in one dim. with unit mass -

Lagrangian $$L = \frac{1}{2}\left(\frac{dq}{dt}\right)^2 - V(q)$$



- Quantum Tunneling
  from the “false vacuum” q_0 to the “true vacuum” σ



* Semiclassical approximation *

classical path in Euclidean time τ (−∞<τ <∞)

→ a particle moving in the potential −V in time τ

Tunneling probability $$\Gamma / V = A e^{-B/\hbar}[1 + O(\hbar)]$$

where $$\bar{S} = B = \int_{-\infty}^{\infty} d\tau L_E(q(\tau)) = \int d\tau \left[\frac{1}{2}\left(\frac{dq}{d\tau}\right)^2 + V(q)\right]$$ = Classical Euclidean action (difference)

Eq. of motion : $\tau = it$

$$\frac{d^2 q}{d\tau^2} + \left(-\frac{dV}{dq}\right) = 0$$

boundary conditions

$$x_i = x_f = 0$$

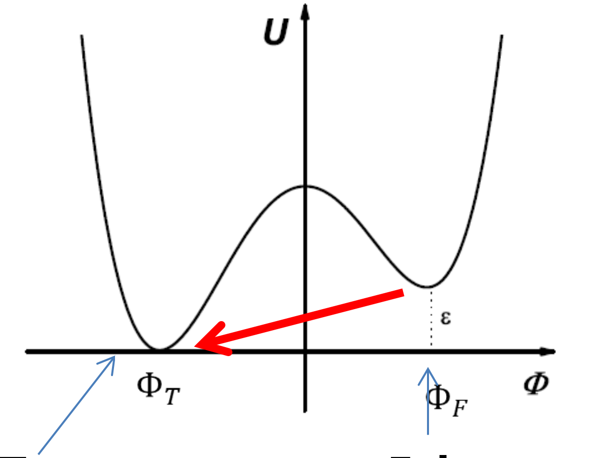
$$\lim_{\tau \to \mp\infty} q(\tau) = q_o, \quad \frac{dq}{d\eta}\Big|_{\tau = 0(\sigma)} = 0$$

- Evolution after tunneling (at τ = 0=t) : Propagation in Minkowski time, t > 0)

(2) Tunneling in field theory (in flat spacetime – no gravity)

Theory with single scalar field

$$S=\int\sqrt{-\eta}\,d^4x\left[-\frac{1}{2}\nabla^{\alpha}\Phi\nabla_{\alpha}\Phi-U(\Phi)\right]$$



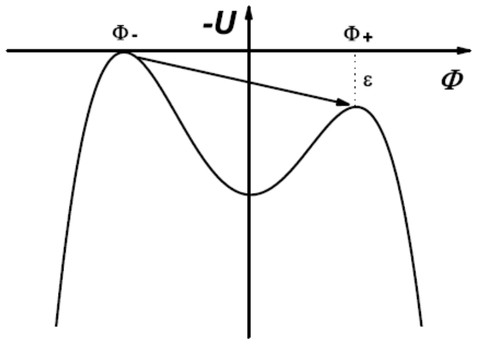
Equation for the bounce $\delta\int d\tau L_E^b=0$

$$\left(\frac{\partial^2}{\partial\tau^2}+\nabla^2\right)\Phi=U'(\Phi)$$

with boundary conditions (Finite size bubble)

$$\lim_{\tau\to\pm\infty}\Phi(\tau,\vec{x})=\Phi_F \qquad \lim_{|\vec{x}|\to\infty}\Phi(\tau,\vec{x})=\Phi_F$$

$$\frac{\partial\Phi}{\partial\tau}(0,\vec{x})=0$$

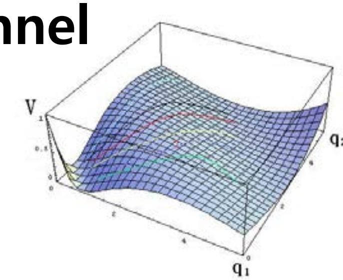


Tunneling rate :

$$\Gamma/V=Ae^{-B/\hbar}[1+O(\hbar)] \qquad B=S_E=\int d\tau d^3x\left[\frac{1}{2}\left(\frac{\partial\Phi}{\partial\tau}\right)^2+\frac{1}{2}(\vec{\nabla}\Phi)^2+U(\Phi)\right]$$

Note : Multidim. has many min. (codim=1) & paths to tunnel

The leading tunneling is from the path and endpoint that minimize the tunneling exponent B.



O(4)-symmetry : Rotationally invariant Euclidean metric

$$ds^2 = d\eta^2 + \eta^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)] \qquad (\eta^2 = \tau^2 + r^2)$$

Tunneling probability factor

$$B = S_E = 2\pi^2 \int_0^\infty \eta^3 d\eta \left[\frac{1}{2}\left(\frac{\partial\Phi}{\partial\eta}\right)^2 + U(\Phi)\right]$$

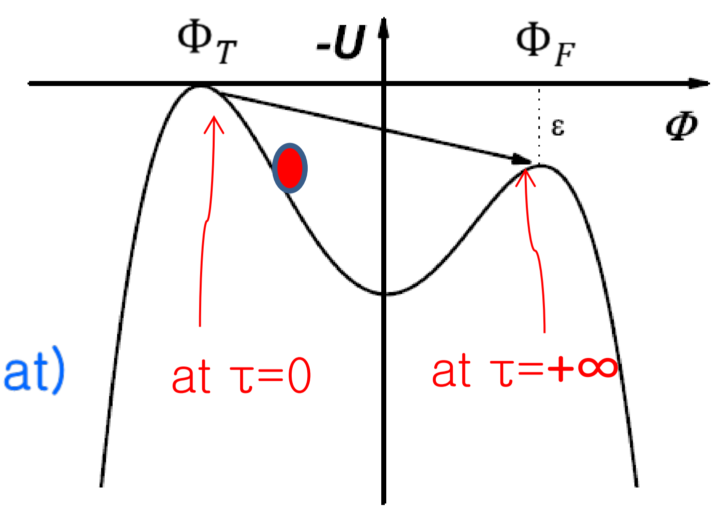
The Euclidean field equations & boundary conditions $\left(\frac{\partial^2}{\partial\tau^2} + \nabla^2 = \frac{d^2}{d\eta^2} + \frac{3}{\eta}\frac{d}{d\eta}\right)$

$$\Phi'' + \frac{3}{\eta}\Phi' = \frac{dU}{d\Phi} \qquad \lim_{\eta\to\infty}\Phi(\eta) = \Phi_F, \frac{d\Phi}{d\eta}|_{\eta=0} = 0$$

"Particle" Analogy : (at position Φ at time η)

The motion of a particle in the potential –U with the damping force proportional to 1/ η

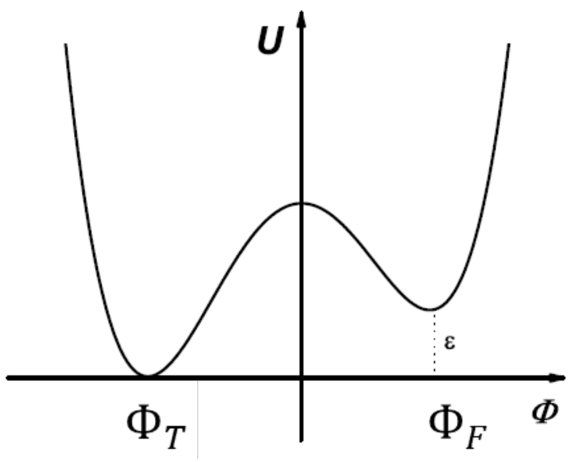
At time 0,
the particle is released at rest
(The initial position should be chosen such that)
at time infinity,
the particle will come to rest at Φ_F .

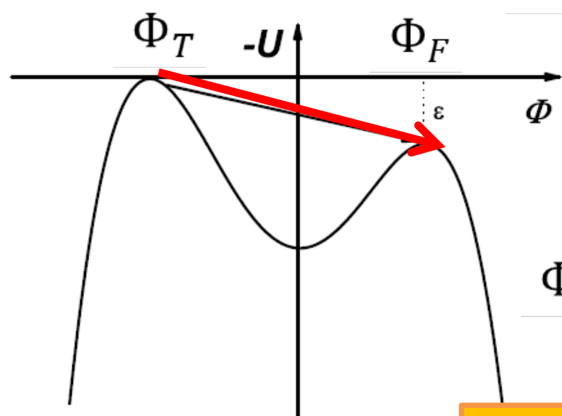


Thin-wall approximation

$$U(\Phi) = \frac{\lambda}{8}(\Phi^2 - b^2)^2 - \frac{\varepsilon}{2b}(\Phi - b)$$

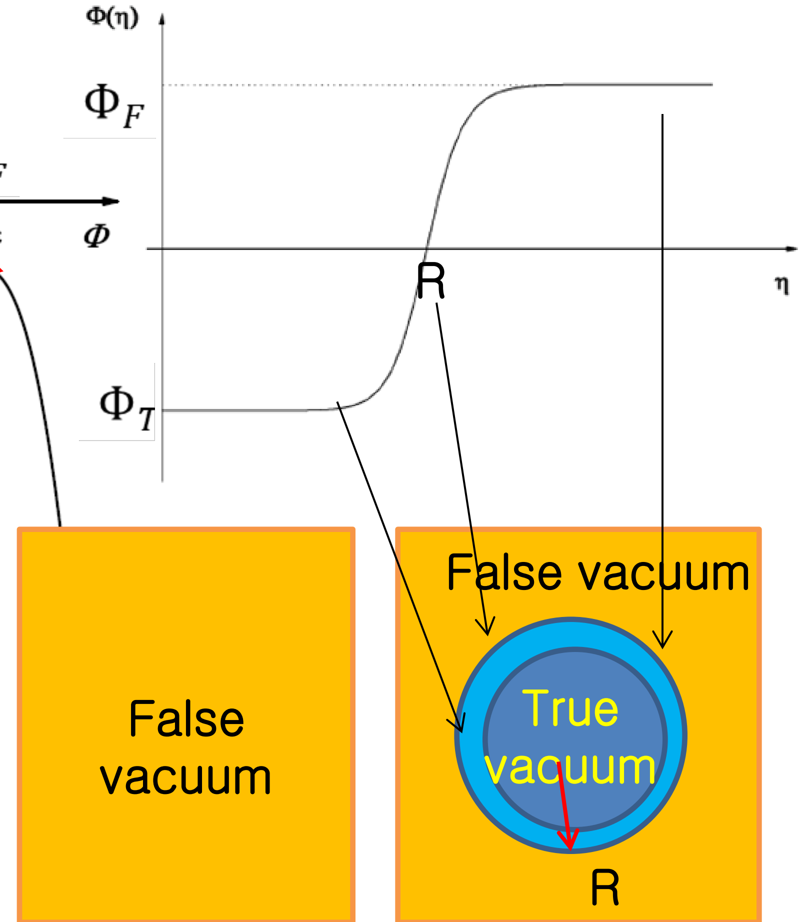
(ε: small parameter)





Bubble solution

Large 4dim. spherical bubble with radius R and thin wall



B is the difference $B = S_E^b - S_E^F$

$$B = B_{in} + B_{wall} + B_{out}$$

Inside the wall $B_{in} = -\frac{1}{2}\pi^2\eta^4\varepsilon$

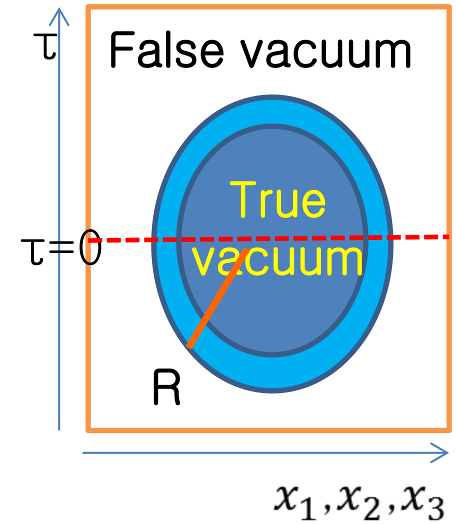
Outside the wall $B_{out} = S_E(\Phi_F) - S_E(\Phi_F) = 0$

on the wall $B_{wall} = 2\pi^2\eta^3 S_o,$ where $S_o = \int_{\Phi_T}^{\Phi_F}\sqrt{2[U(\Phi) - U(\Phi_T)]}\,d\Phi$

the radius of a true vacuum bubble $\eta = 3S_o/\varepsilon \equiv \mathrm{R}$

the nucleation rate of a true vacuum bubble $B_o = S_E = \frac{27\pi^2 S_o^4}{2\varepsilon^3}$

Evolution of the bubble

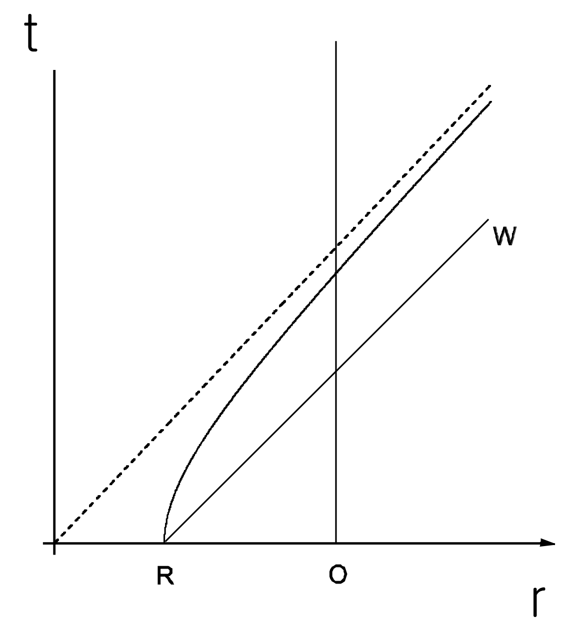


The false vacuum makes a quantum tunneling into a true vacuum bubble at time τ=t=0, such that

$$\Phi(t=0, \vec{x}) = \Phi(\tau=0, \vec{x})$$

$$\frac{\partial}{\partial t}\Phi(t=0, \vec{x}) = 0.$$

Afterwards, it evolves in Lorentzian spacetime.



$$-\frac{\partial^2 \Phi}{\partial^2 t} + \nabla^2 \Phi = U'(\Phi)$$

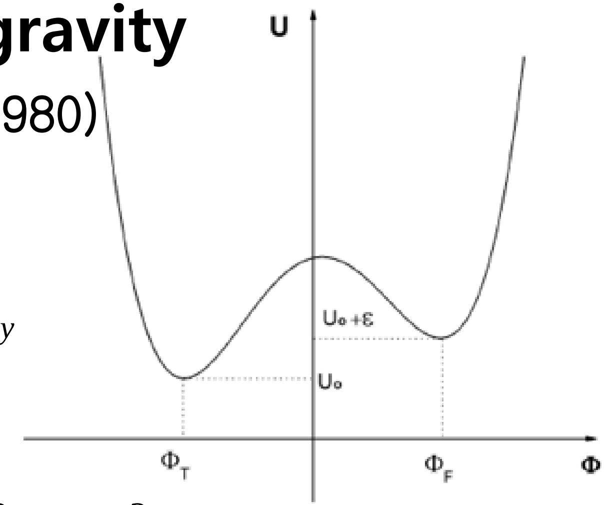
The solution (by analytic continuation)

$$\Phi(t, \vec{x}) = \Phi(\eta = (|\vec{x}|^2 - t^2)^{1/2})$$

(Note: Φ is a function of η) $(\eta^2 = \tau^2 + r^2)$

2.2 Bubble nucleation in the Einstein gravity

S. Coleman and F. De Luccia, PRD21, 3305 (1980)



Action

$$S = \int \sqrt{g} d^4x \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi - U(\Phi) \right] + S_{boundary}$$

O(4)-symmetric Euclidean metric Ansatz

$$ds^2 = d\eta^2 + \rho^2(\eta)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$$

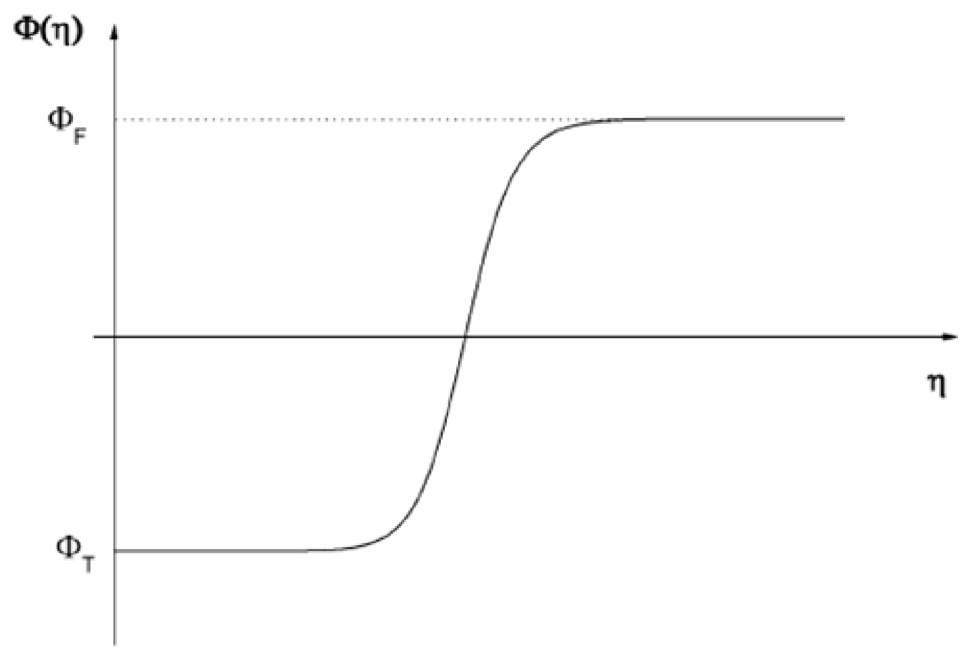
The Euclidean field equations (scalar eq. & Einstein eq.)

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = \frac{dU}{d\Phi}$$

$$\rho'^2 = 1 + \frac{\kappa\rho^2}{3}\left(\frac{1}{2}\Phi'^2 - U\right)$$

boundary conditions for bubbles

$$\lim_{\eta \to \eta(\max)} \Phi(\eta) = \Phi_F, \frac{d\Phi}{d\eta}\Big|_{\eta=0} = 0$$

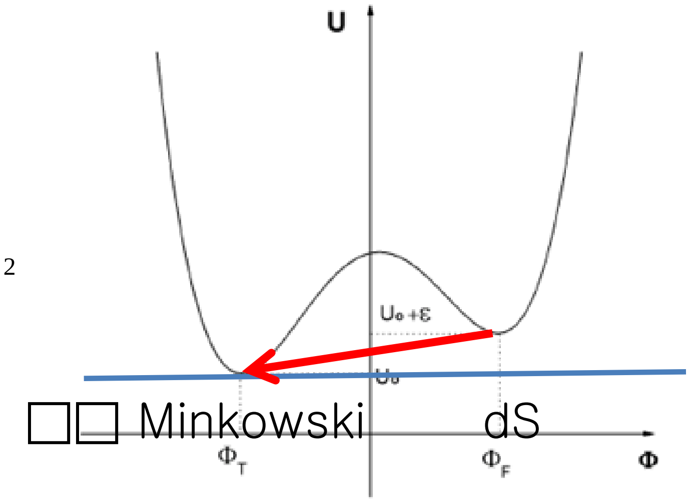


(i) From de Sitter to flat spacetime

the radius of the bubble $$\bar{\rho} = \frac{\bar{\eta}}{1+(\bar{\eta}/2\Lambda)^2}$$

where $$\Lambda = (\kappa\varepsilon/3)^{-1/2}$$

the nucleation rate $$B = \frac{B_o}{[1+(\bar{\eta}/2\Lambda)^2]^2}$$



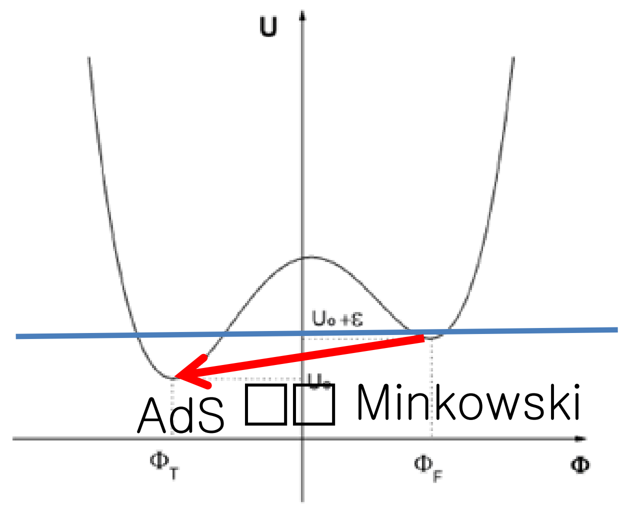
Note : 1) $\bar{\rho} < \bar{\eta}$ gravity makes the bubble smaller in dS

2) $B < B_0$ Transition probability increases

(ii) From flat to Anti-de Sitter spacetime

the radius of the bubble $$\bar{\rho} = \frac{\bar{\eta}}{1-(\bar{\eta}/2\Lambda)^2}$$

the nucleation rate $$B = \frac{B_o}{[1-(\bar{\eta}/2\Lambda)^2]^2}$$



Note : $\bar{\rho} > \bar{\eta}$ the bubble becomes larger

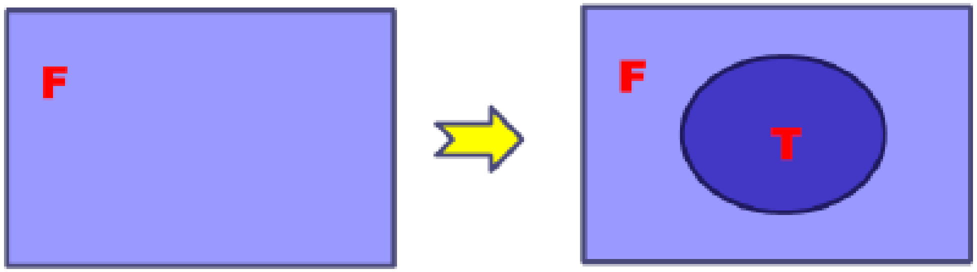
$B > B_0$ Transition probability decreases.

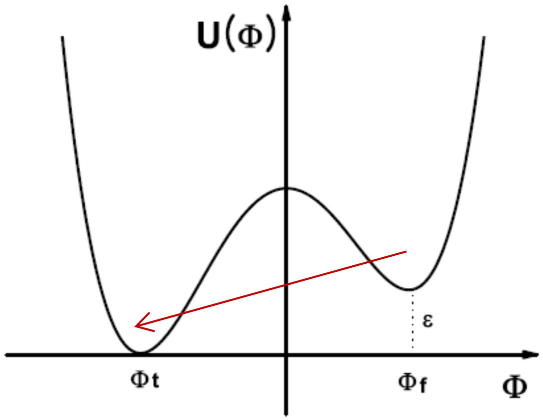
For small enough ϵ, false vacuum can be stable

Homogeneous tunneling

A channel of the inhomogeneous tunneling

before after

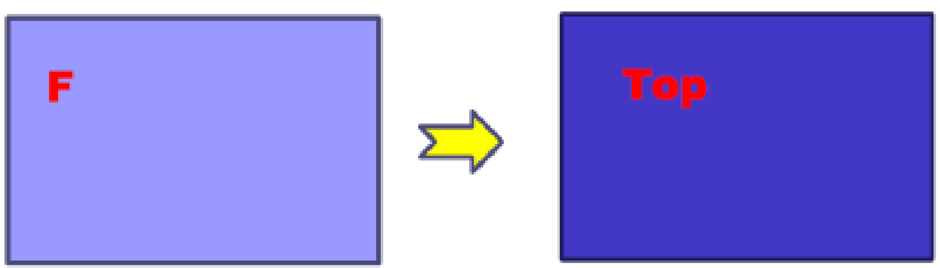


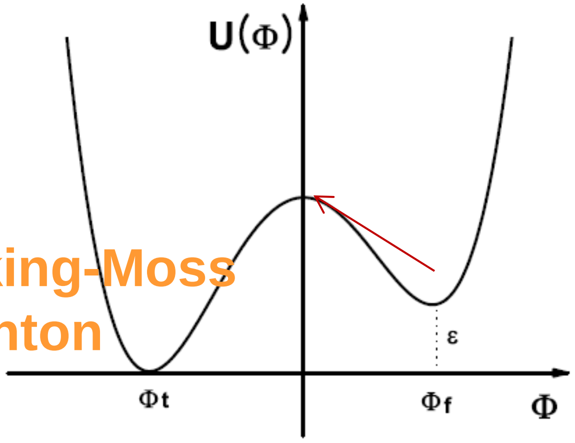


Ordinary bounce (vacuum bubble) solutions

A channel of the homogeneous tunneling

before after





Hawking-Moss instanton

3. More on Bubbles and Tunneling

3.1 False vacuum bubble nucleation

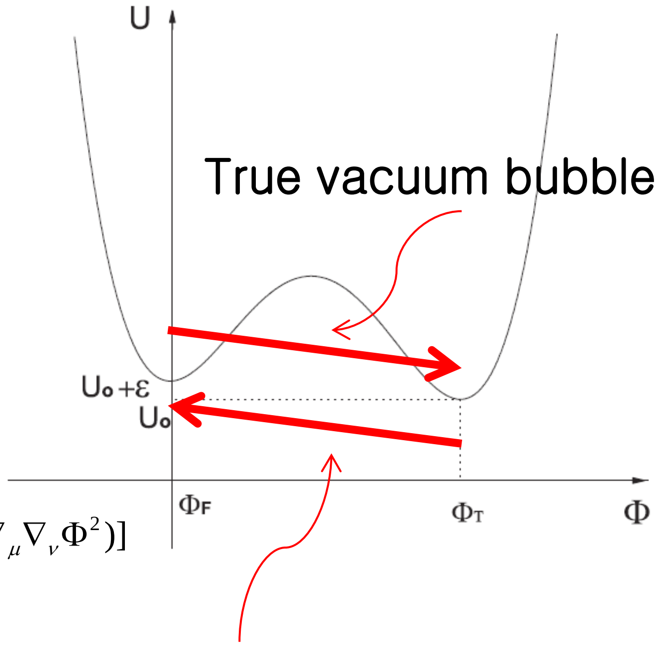
– The Einstein theory of gravity with a nonminimally coupled scalar field

Action

$$S = \int \sqrt{g}\, d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla^\alpha \Phi \nabla_\alpha \Phi - \frac{1}{2}\xi R \Phi^2 - U(\Phi)\right] \quad + S_{boundary}$$

Potential

$$U(\Phi) = \frac{\lambda}{8}\Phi^2(\Phi - 2b)^2 - \frac{\varepsilon}{2b}(\Phi - 2b) + U_o$$



Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{1 - \xi\Phi^2\kappa}\left[\nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu}\left(\frac{1}{2}\nabla^\alpha \Phi \nabla_\alpha \Phi + U\right) + \xi\left(g_{\mu\nu}\nabla^\alpha \nabla_\alpha \Phi^2 - \nabla_\mu \nabla_\nu \Phi^2\right)\right]$$

curvature scalar

$$R = \frac{\kappa[4U(\Phi) + \nabla^\mu \Phi \nabla_\mu \Phi - 3\xi \nabla^\mu \nabla_\mu \Phi^2]}{1 - \xi \Phi^2 \kappa}$$

Rotationally invariant Euclidean metric ：O(4)−symmetry

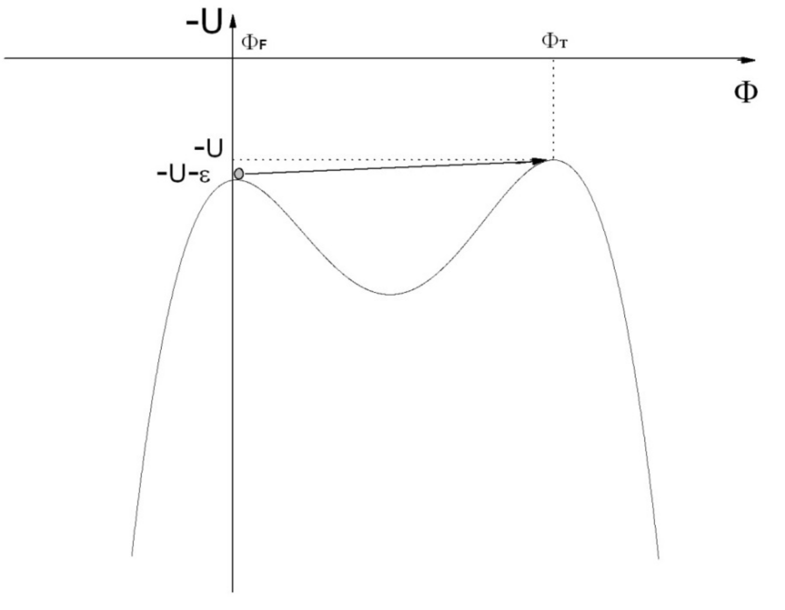
$$ds^2 = d\eta^2 + \rho^2(\eta)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$$

The Euclidean field equations

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' - \xi R_E \Phi = \frac{dU}{d\Phi} \qquad \rho'^2 = 1 + \frac{\kappa\rho^2}{3(1-\xi\Phi^2\kappa)}\left(\frac{1}{2}\Phi'^2 - U\right)$$

boundary conditions

$$\lim_{\eta\to\eta(\max)} \Phi(\eta) = \Phi_T, \frac{d\Phi}{d\eta}\Big|_{\eta=0} = 0$$



Our main idea

$$\xi R_E \Phi > \frac{3\rho'}{\rho}\Phi'$$

(during the phase transition)

True & False Vacuum Bubbles

(*)Lee, Weinberg, PRD

	False-to-true (True vac. Bubble)	True-to-false (*) **(False vac. Bubble)**
De Sitter – de Sitter	O	O (*)
Flat – de Sitter	O	O
Anti de Sitter – de Sitter	O	X
Anti de Sitter – flat	O	O
Anti de Sitter – Anti de Sitter	O	O

(*) exists in

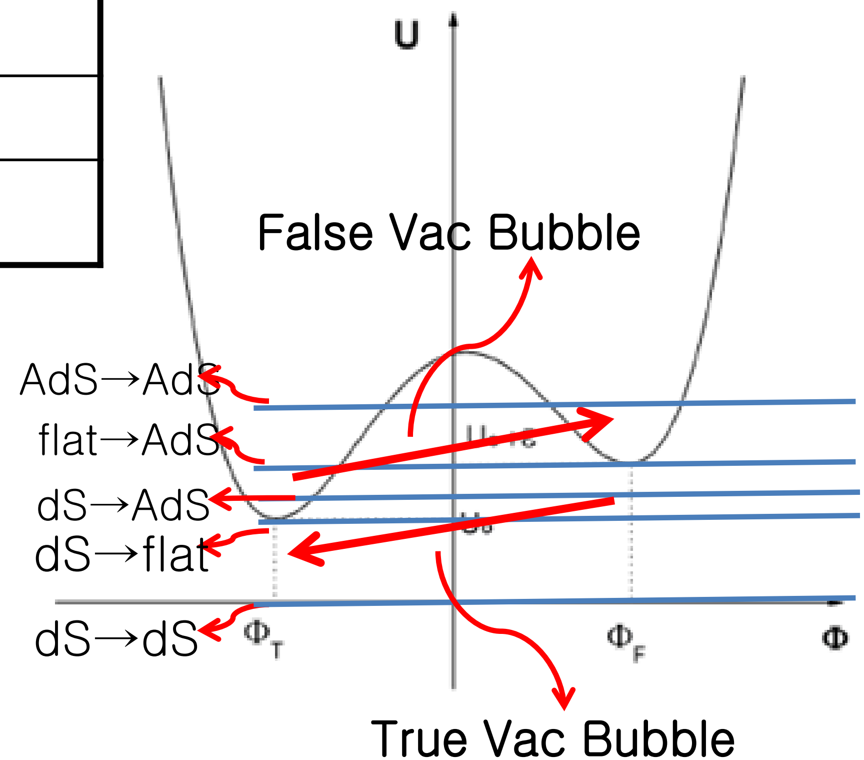
(1)non-minimally coupled gravity

(W.Lee, BHL, C.H.Lee,C.Park, PRD(2006))

or in

(2)Brans-Dicke type theory

(H.Kim,BHL,W.Lee, Y.J. Lee, D.-H.Yoem, PRD(2011))



Dynamics of False Vacuum Bubble :

Can exist an expanding false vac bubble inside the true vacuum

BHL, C.H.Lee, W.Lee, S. Nam, C.Park, PRD(2008) (for nonminimal coupling)

BHL, W.Lee, D.-H. Yeom, JCAP(2011) (for Brans-Dicke)

3.2 vacuum bubbles with finite geometry

BHL, C.H. Lee, W.Lee & C.Oh,

dS-dS

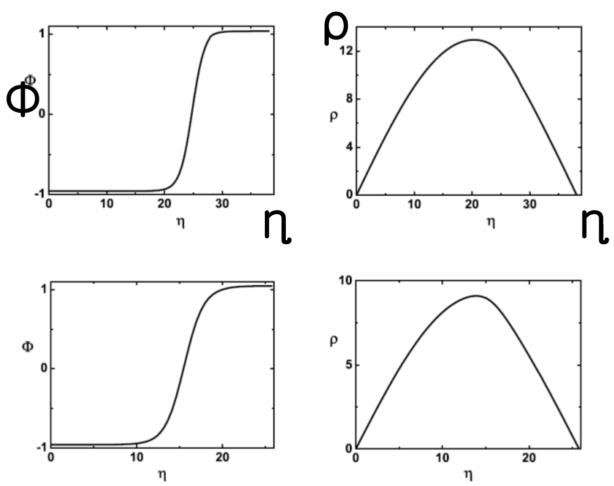


Figure 2: dS-dS cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = 0.1$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.2$, and $U_0 = 0.1$ for for bottom figure.

dS-AdS

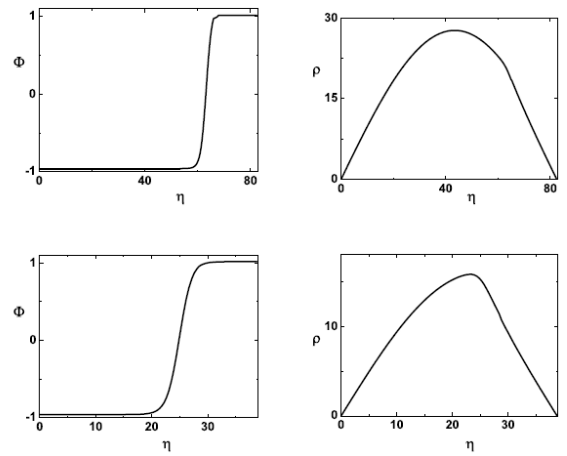


Figure 3: dS-AdS cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = -0.04$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.3$, and $U_0 = -0.04$ for for bottom figure.

dS-flat

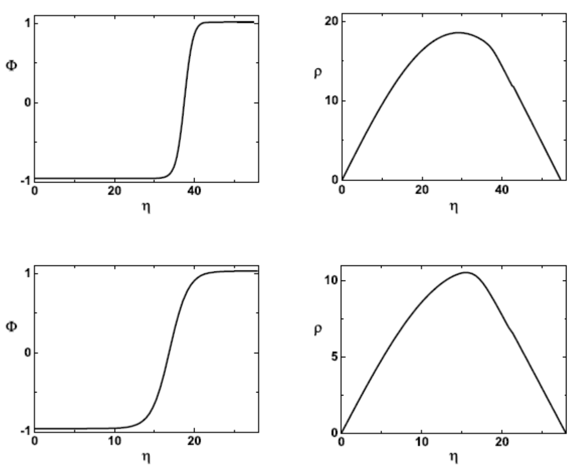


Figure 4: ds-flat cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = 0.0077$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.3$, and $U_0 = 0.0077$ for for bottom figure.

flat-AdS and AdS-AdS

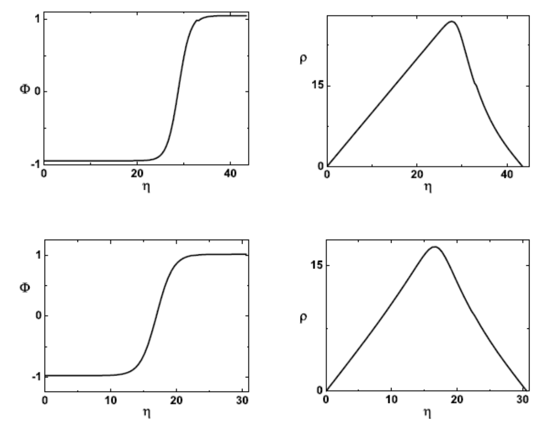
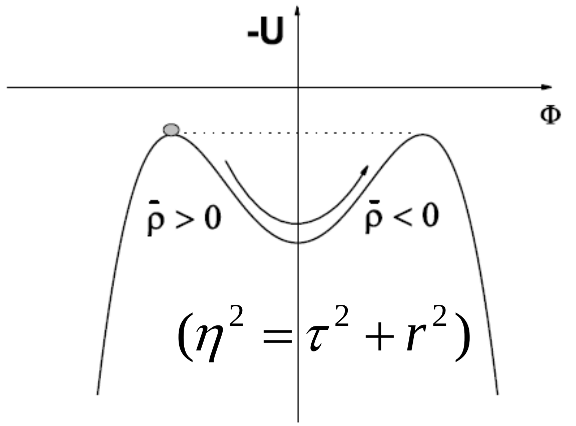


Figure 5: Flat-AdS and AdS-AdS cases. $\epsilon = 0.05$, $\kappa = 0.7$, and $U_0 = -0.09868$ for flat-AdS case. $\epsilon = 0.02$, $\kappa = 0.7$, and $U_0 = -0.05$ for AdS-AdS case.

3.3 Tunneling between the degenerate vacua

∃ Z2−symm. with finite geometry bubble



Potential $U(\Phi) = \frac{\lambda}{8}\left(\Phi^2 - \frac{\mu^2}{\lambda}\right)^2 + U_o.$

O(4)−symmetric Euclidean metric

$$ds^2 = d\eta^2 + \rho^2(\eta)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$$

Equations of motions

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = \frac{dU}{d\Phi} \qquad \rho'' = -\frac{\kappa}{3}\rho(\Phi'^2 + U),$$

Boundary condition (consistent with Z2−sym.)

$$\rho|_{\eta=0} = 0, \quad \rho|_{\eta=\eta_{max}} = 0, \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=0} = 0, \quad \text{and} \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=\eta_{max}} = 0.$$

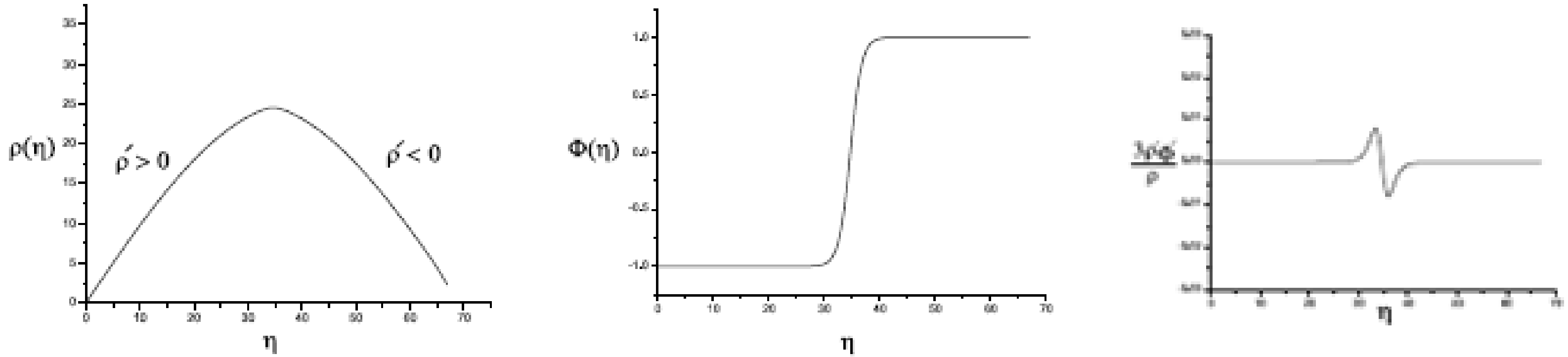
− in de Sitter space.

The numerical solution by Hackworth and Weinberg.

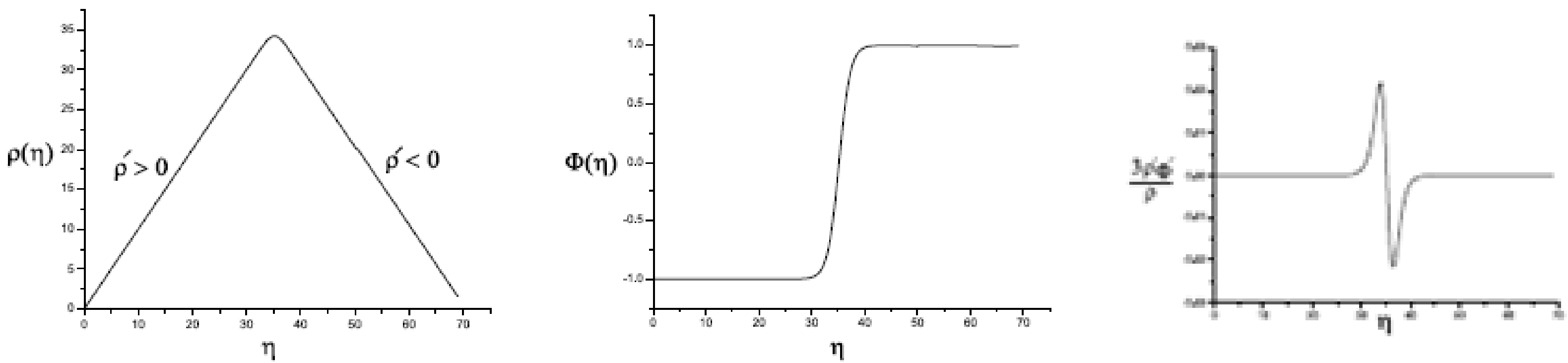
The analytic computation and interpretation :

(BHL & W. Lee, CQG (2009))

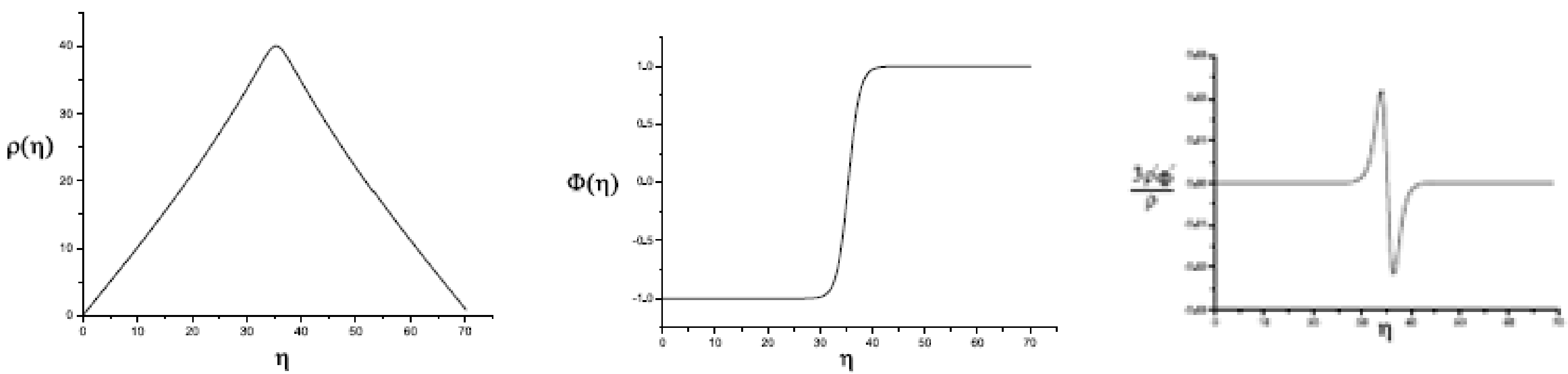
dS − dS



flat −flat



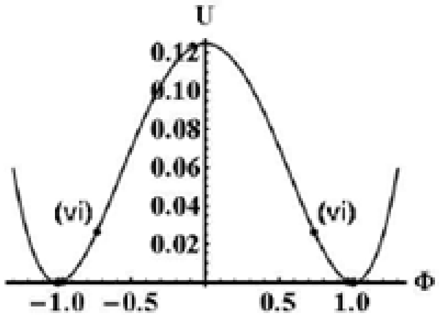
AdS−AdS



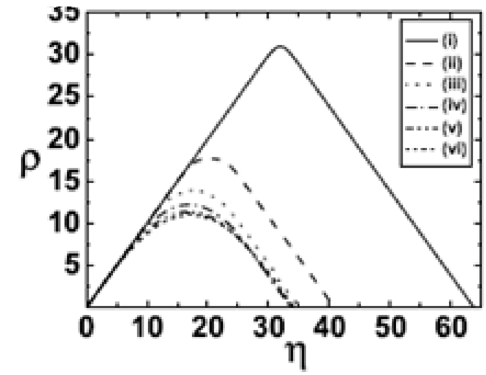
B.-H. L, C. H. Lee, W. Lee & C. Oh, PRD**82** (2010)

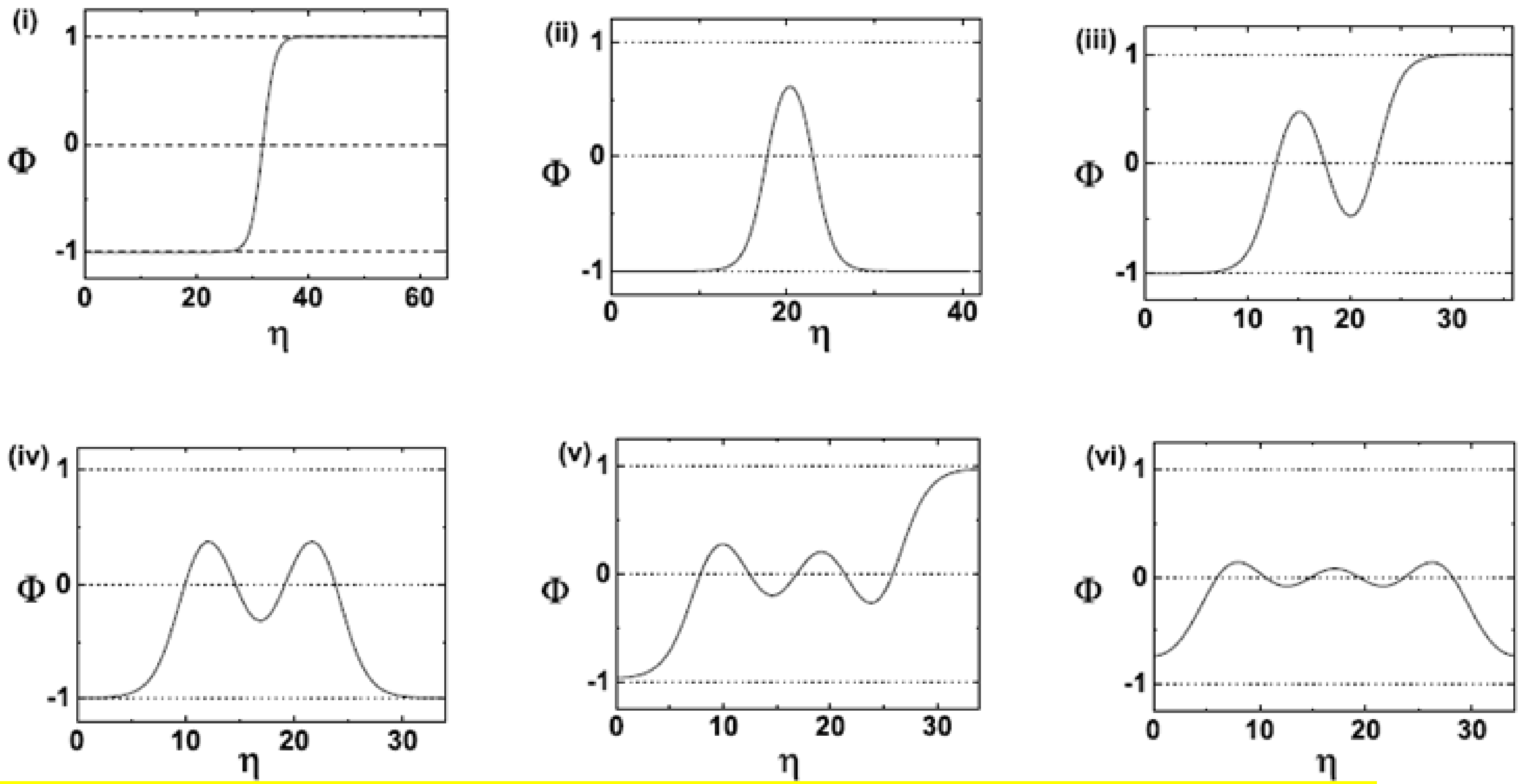
3.3 Oscillaing solutions : a) between flat-flat degenerate vacua ($\tilde{\kappa} = 0.2$)

B.-H. Lee, C. H. Lee, W. Lee & C. Oh, arXiv:1106.5865



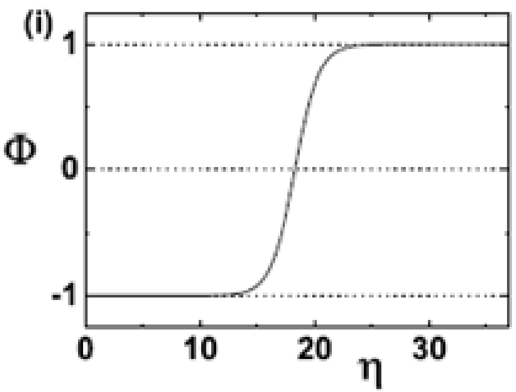
Number of Oscillation	Φ_0
1	-0.99999999999355985
2	-0.99999585754
3	-0.9995857315805
4	-0.994499
5	-0.9661682
6	-0.7348584

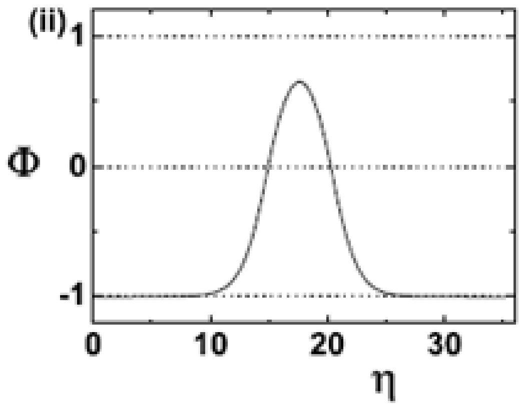


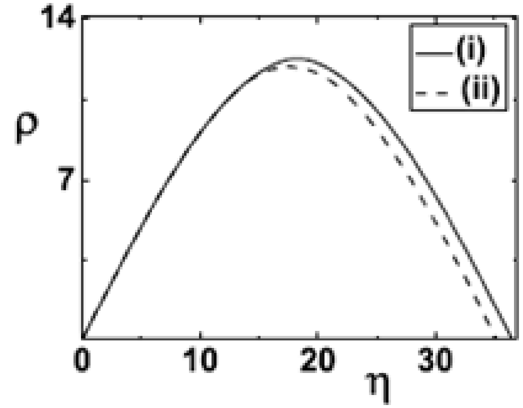


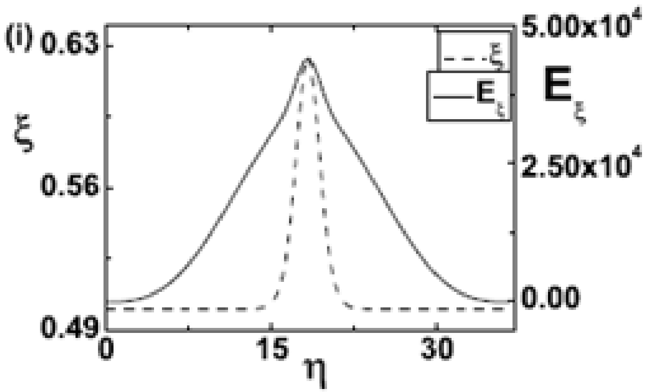
This type of solutions is possible only if gravity is taken into account.

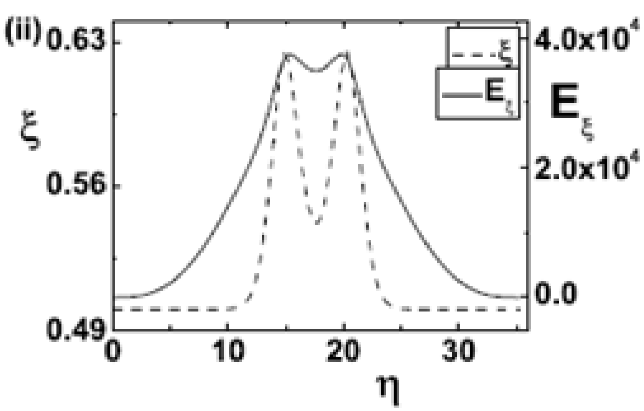
3.3 Oscillaing solutions - b) between dS-dS degenerate vacua







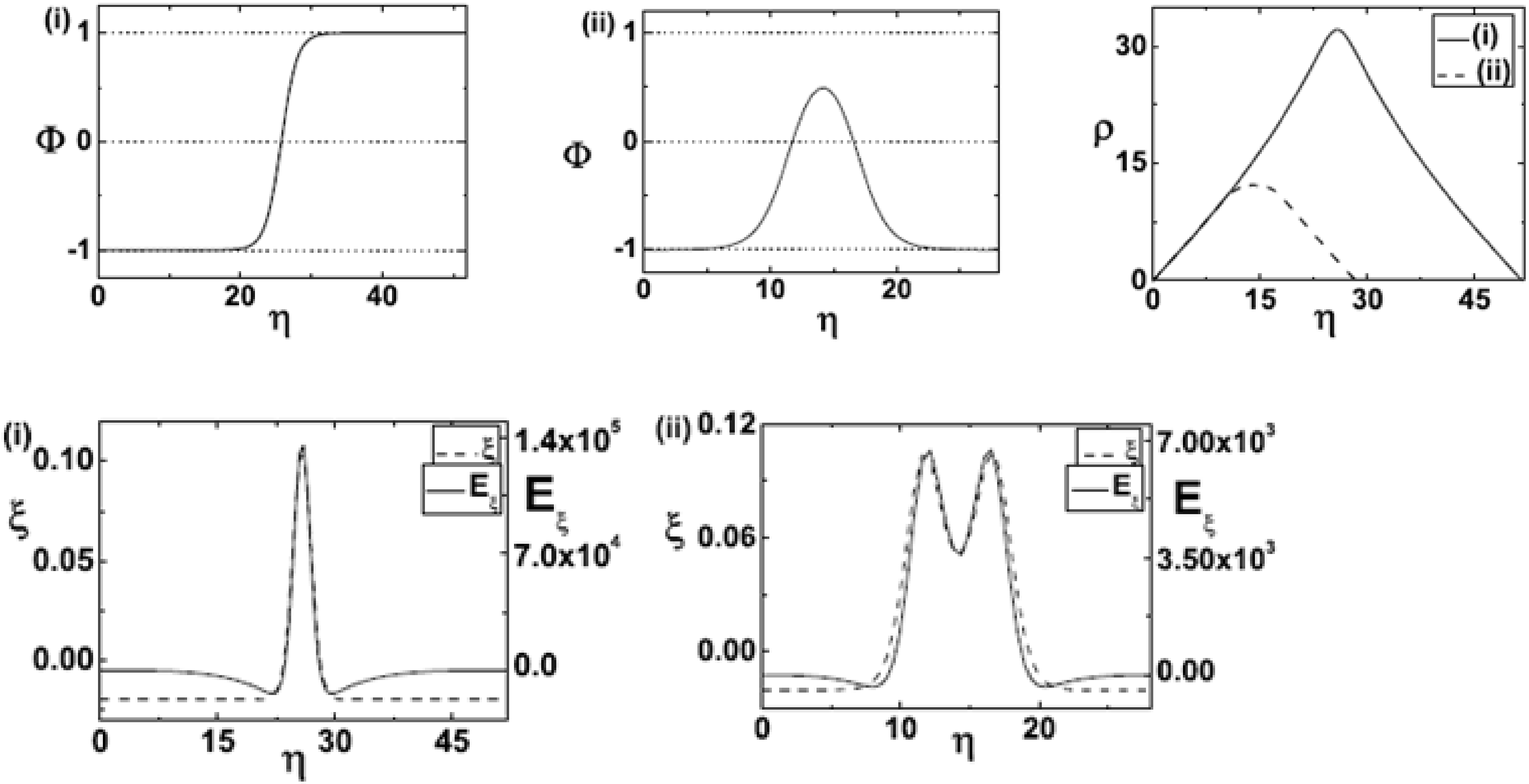




$\tilde{U}_o = 0.5$ and $\tilde{\kappa} = 0.04$

3.3 Oscillaing solutions – c) between AdS-AdS degenerate vacua

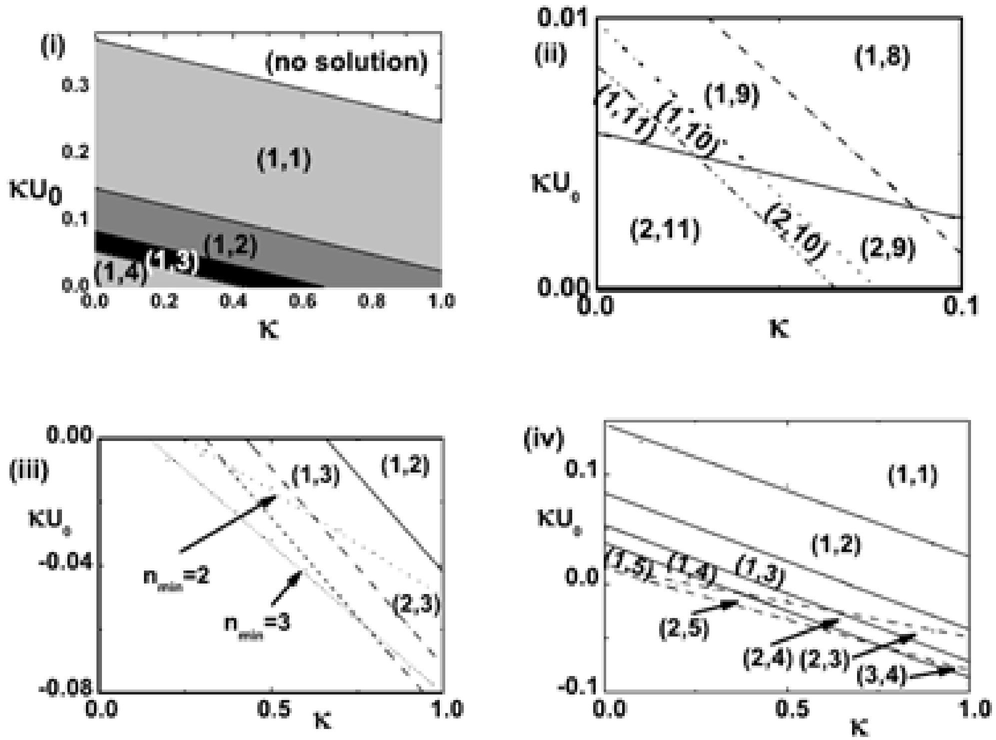
$\tilde{U}_o = -0$and $\tilde{\kappa} = 0.4$

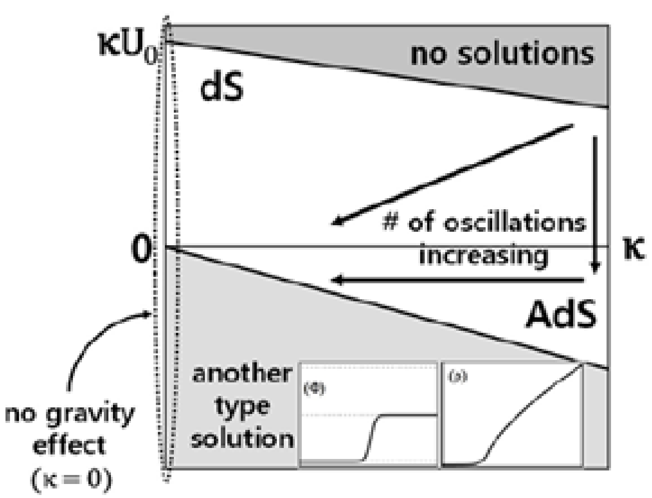


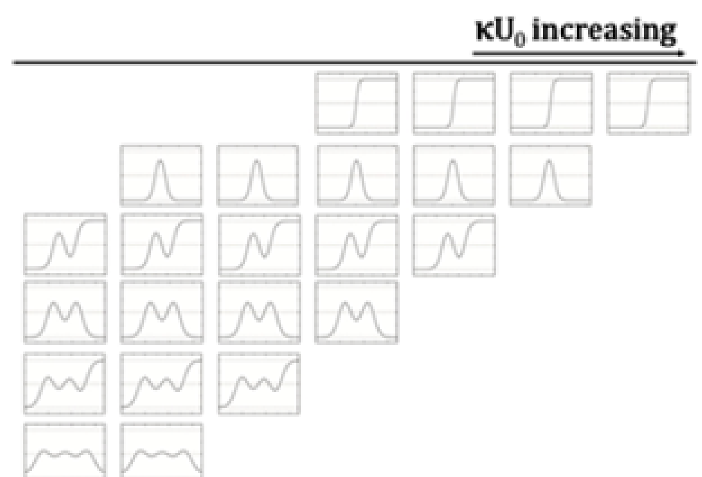
The phase space of solutions

$0 \leqq \tilde{\kappa} \leqq 1$

the notation (n_min, n_max), where n_min means the minimum number of oscillations and n_max the maximum number of oscillation







3.4 Fubini Instanton in Gravity

Review : In the Absence of Gravity

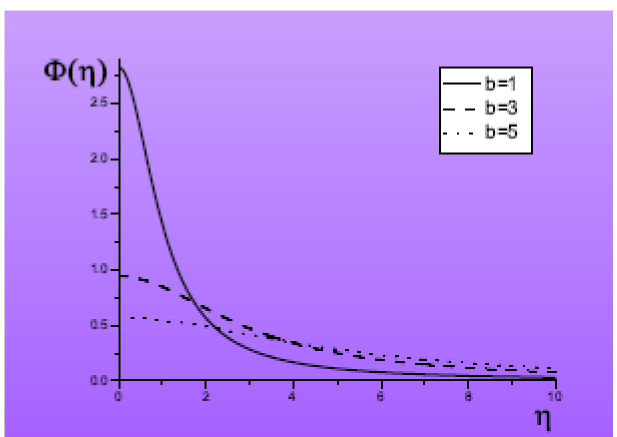
action
$$S = \int_{\mathcal{M}} \sqrt{-g} d^4x \left[-\frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi - U(\Phi) \right]$$

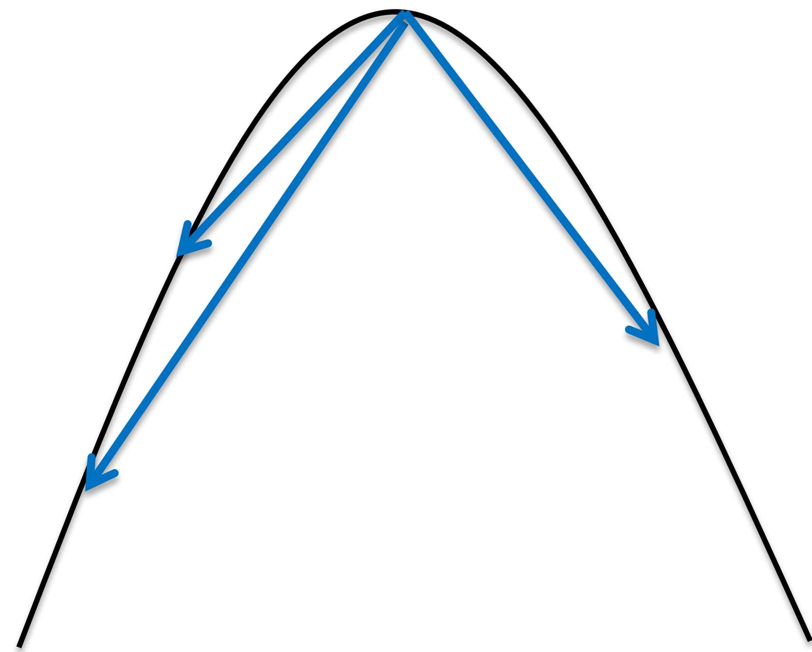
$$\boldsymbol{U(\Phi) = -\frac{\lambda}{4}\Phi^4}$$

Equation of motion
$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = \frac{dU}{d\Phi}$$

Boundary conditions
$$\Phi|_{\eta=0} = \Phi_o \quad \text{and} \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=\infty} = 0$$

solution
$$\Phi(\eta) = \sqrt{\frac{8}{\lambda}} \frac{b}{\eta^2 + b^2}$$





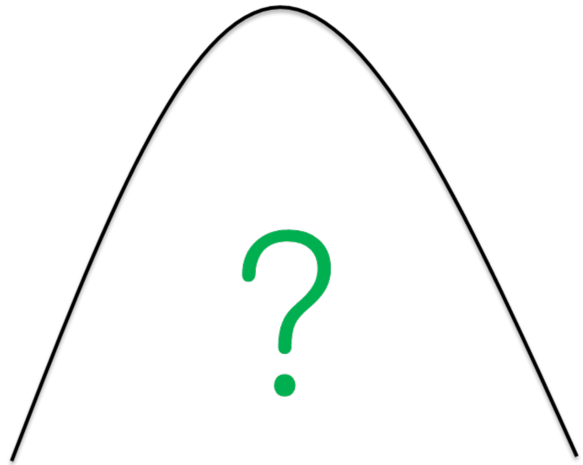
- Fubini, Nuovo Cimento 34A (1976)
- Lipatov – JETP45 (1977)

In the Presence of Gravity

$$U(\Phi) = \frac{\lambda}{4}\Phi^4 - \frac{m^2}{2}\Phi^2 + \epsilon\Phi$$

$$U(\Phi) = -\frac{\lambda}{4}\Phi^4$$

$$U(\Phi) = -\frac{\lambda}{4}\Phi^4 + \frac{m^2}{2}\Phi^2$$



Action

$$S = \int_{\mathcal{M}} \sqrt{-g}\, d^4x \left[-\frac{1}{2}\nabla^\alpha\Phi\nabla_\alpha\Phi - U(\Phi) \right]$$

O(4) symmetric metric

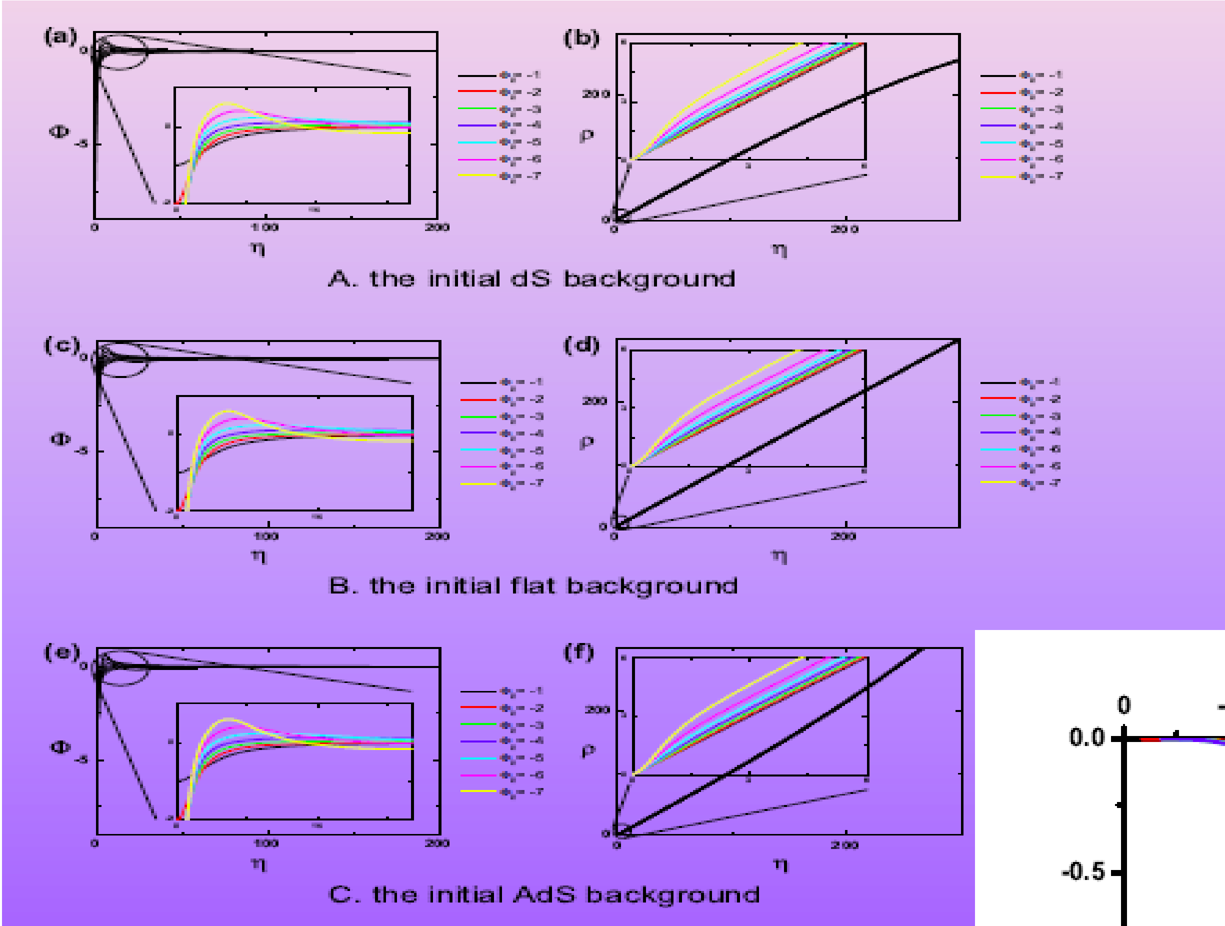
$$ds^2 = d\eta^2 + \rho(\eta)^2 \left[d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right]$$

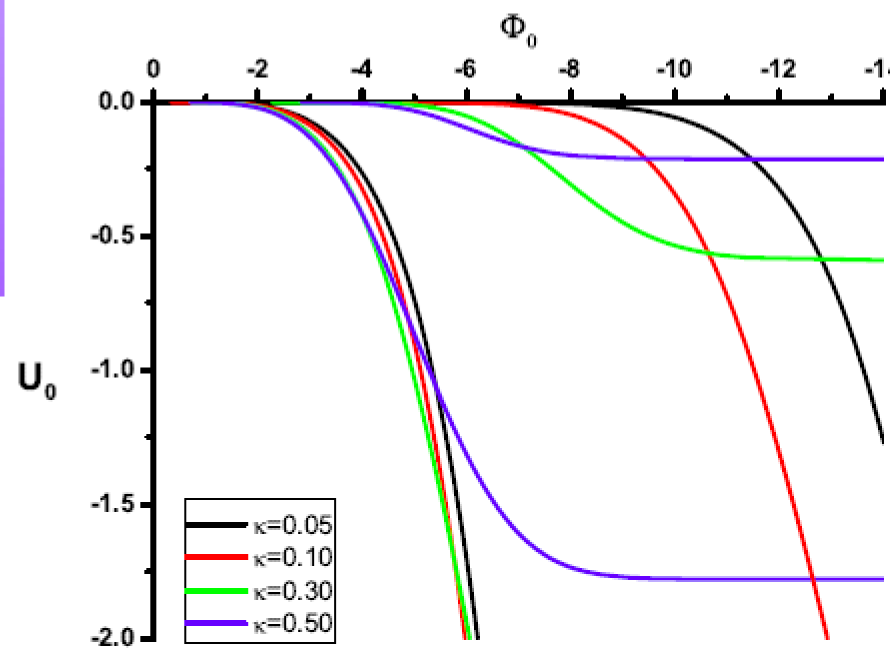
Equantions of motion

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = \frac{dU}{d\Phi} \qquad \rho'' = -\frac{\kappa}{3}\rho\left(\Phi'^2 + U\right) \qquad \rho'^2 - 1 - \frac{\kappa\rho^2}{3}\left(\frac{1}{2}\Phi'^2 - U\right) = 0,$$

Boundary Conditions

$$\rho|_{\eta=0} = 0, \quad \left.\frac{d\rho}{d\eta}\right|_{\eta=0} = 1, \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=0} = 0, \quad \text{and} \quad \Phi|_{\eta=\eta_{max}} = 0$$





B-HL, W. Lee, C.Oh, D. Yeom, D. Rho
JHEP 1306 (2013) 003;
in preparation

Figure 9: Phase diagram for $\Lambda \leq 0$ with several κ's.

4. Possible Cosmological Implication

4.1 5Dim. Z2 symmetric Black hole with a domain wall solution.

After the nucleation, the domain wall (that may be interpreted as our braneworld universe) evolves in the radial direction of the bulk spacetime.

$$r = a(\tau), \quad \dot{a}^2 + V(a) = 0$$

The equation becomes

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{1}{3}\lambda + \frac{2m_*}{a^4} - \frac{q^2}{a^6},$$

$\lambda = 3A$: the effective cosmological constant.
mass term ~ the radiation in the universe
charge term ~ the stiff matter with a negative energy density.

Cosmological solutions

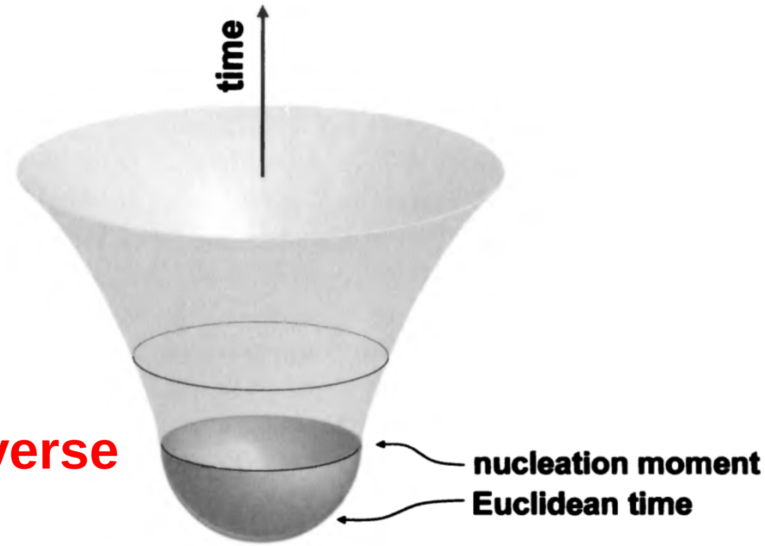
the expanding domain wall (universe) solution ($a > r*$,+).
approaching the de Sitter inflation with λ, since the contributions of the mass and charge terms are diluted.

contracting solution ($a < r*$,+) : the initially collapsing universe.
The domain wall does not run into the singularity & experiences a bounce since there is the barrier in $V(a)$ because of the charge q.

4.2 Application to the No-boundary

before **after**

nothing ⇒ **Something, homogeneous**



to avoid the singularity problem of a Universe

the no-boundary proposal by Hartle and Hawking

cf) Vilenkin's tunneling boundary condition

the ground state wave function of the universe is given by the Euclidean path integral satisfying the WD equation

$$\Psi[h_{\mu\nu},\chi] = \int_{\partial g = h, \partial\phi = \chi} \mathcal{D}g\mathcal{D}\phi \, e^{-S_{\rm E}[g,\phi]}$$

Consider the Euclidean action $S_{\rm E} = -\int d^4x\sqrt{+g}\left(\frac{1}{16\pi}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$

the mini-superspace approximation => the scale factor as the only dof.

$$ds_{\rm E}^2 = d\eta^2 + \rho^2(\eta)\left(d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right)$$

The equations of motion

$$\ddot{\phi} = -3\frac{\dot{\rho}}{\rho}\dot{\phi} \pm V' \,, \qquad \ddot{\rho} = -\frac{8\pi}{3}\rho(\dot{\phi}^2 \pm V) \,,$$

the regular initial conditions at $\eta = 0$

$$\phi = \phi_0 \,, \quad \rho(0) = 0 \,, \quad \dot{\rho}(0) = 1 \,, \quad \dot{\phi}(0) = 0 \,,$$

We impose the followings at the tunneling point $\eta = \eta_{\mathrm{max}}$,

$$\rho(t=0) = \rho(\eta = \eta_{\mathrm{max}}) \,, \quad \dot{\rho}(t=0) = i\dot{\rho}(\eta = \eta_{\mathrm{max}}) \,,$$

$$\phi(t=0) = \phi(\eta = \eta_{\mathrm{max}}) \,, \quad \dot{\phi}(t=0) = i\dot{\phi}(\eta = \eta_{\mathrm{max}}) \,,$$

to analytically continue the solution to the Lorentzian manifold using $d\eta = i dt.$

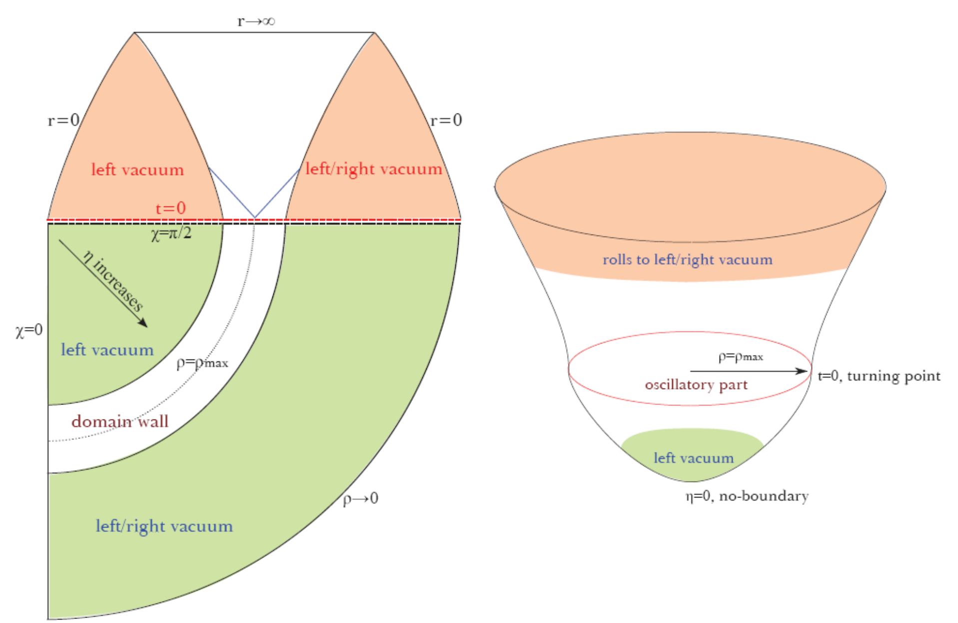


Fig. 2. Left: analytic continuation $\chi \to \pi/2 + it$. Right: analytic continuation $\eta \to \eta_{\mathrm{max}} + it$.

4.3 General vacuum decay problem

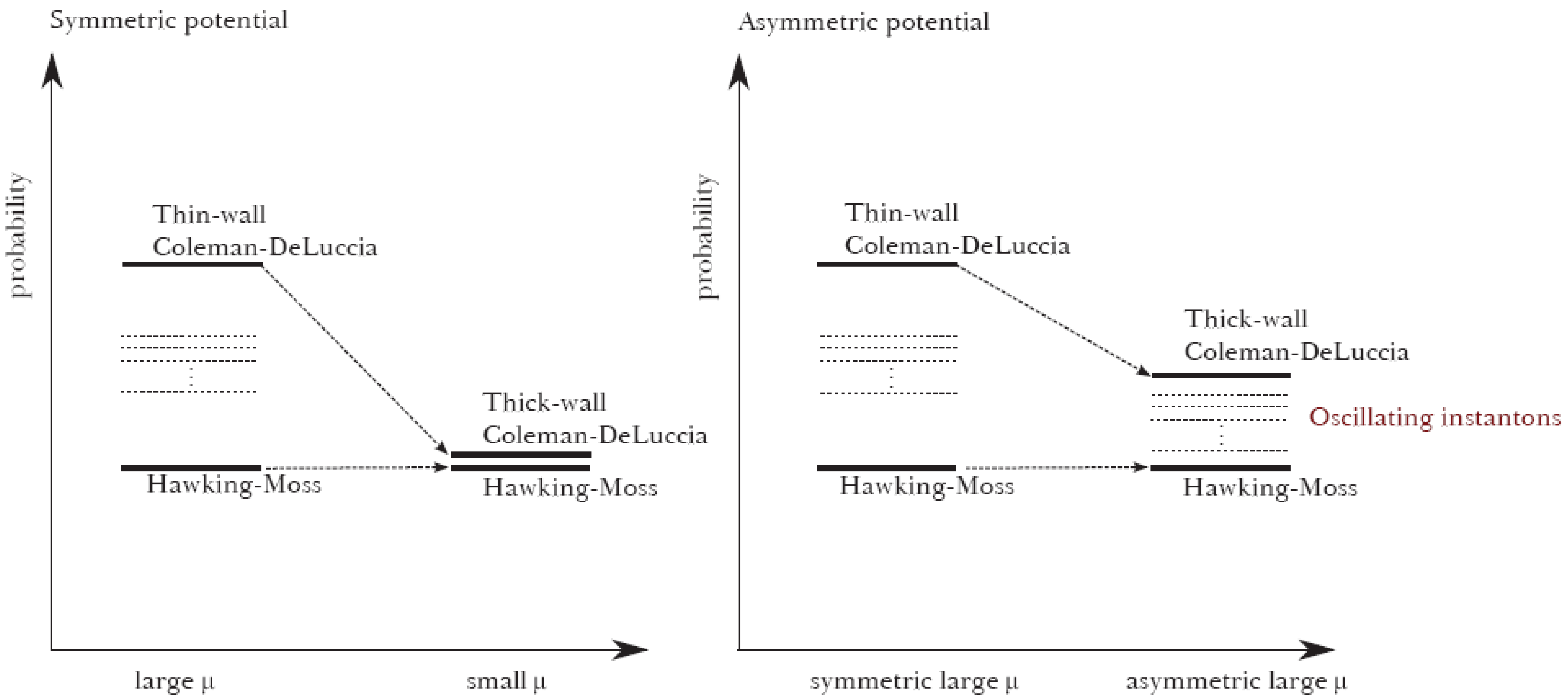


Fig. 9. Conceptual picture of probabilities. Left: When we decrease μ around the local maximum with symmetry, then the thin-wall Coleman–de Luccia solution approaches the thick-wall Coleman–de Luccia solution and this approaches the Hawking–Moss solution. Right: When we change the symmetry with a constant large μ, then the thin-wall Coleman–de Luccia solution approaches a think-wall Coleman–de Luccia solution and oscillating instantons do not disappear.

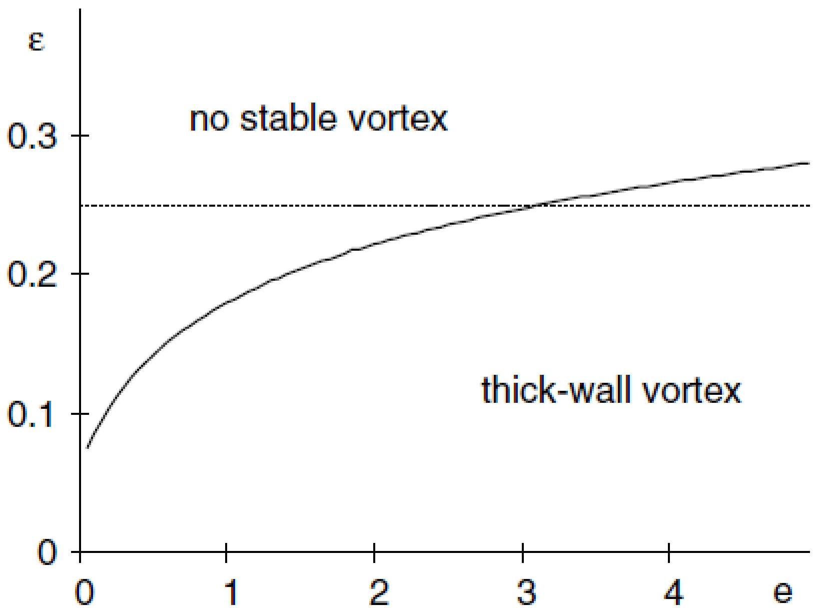
4.4 False Cosmic String and its Decay

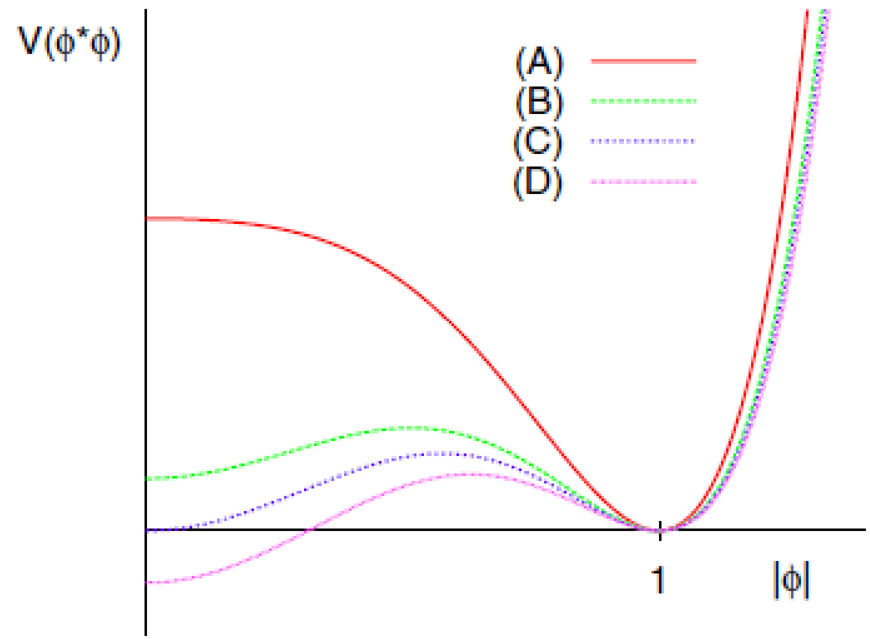
Acton

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi)$$

$$V(\phi^*\phi) = \lambda(|\phi|^2 - \epsilon v^2)(|\phi|^2 - v^2)^2$$

Vortex Solution





B-HL, W. Lee, R. MacKenzie, M.Paranjape U. Yajnik, D-h Yeom.
PRD88 (2013) 085031 arXiv:1308.3501
PRD88 (2013) 105008 arXiv:1310.3005

Decay of the False String (thin wall approximation)

$$S_E = \frac{1}{\lambda v^2}\int d^2x \frac{1}{2} M(R(z,\tau))(\dot{R}^2 + R'^2) + E(R(z,\tau)) - E(R_0)$$

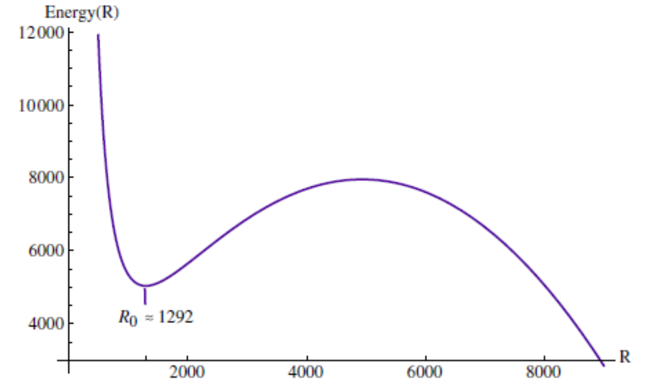


FIG. 1 (color online). The energy as a function of R, for $n = 100$, $e = 0.005$ and $\epsilon = 0.0001$.

Tunnelling Solution

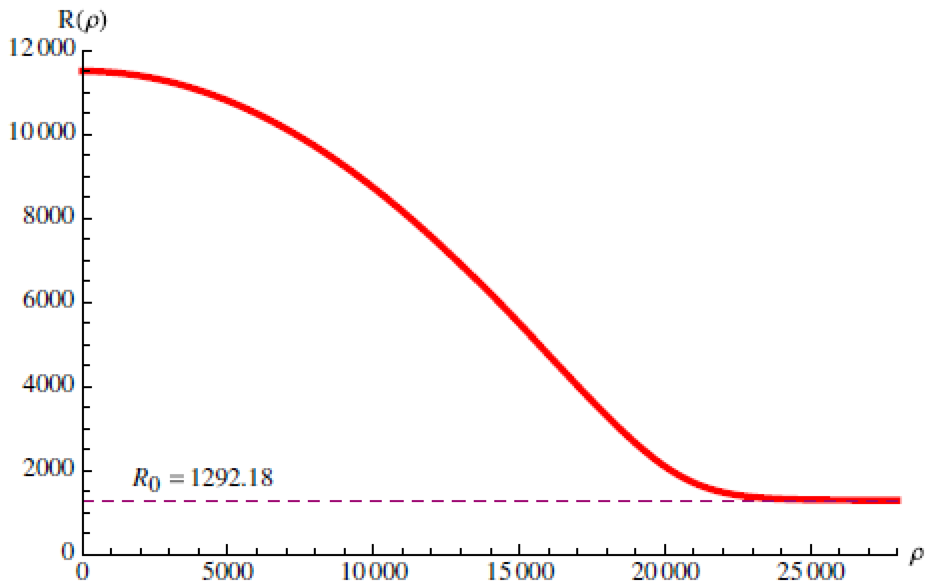
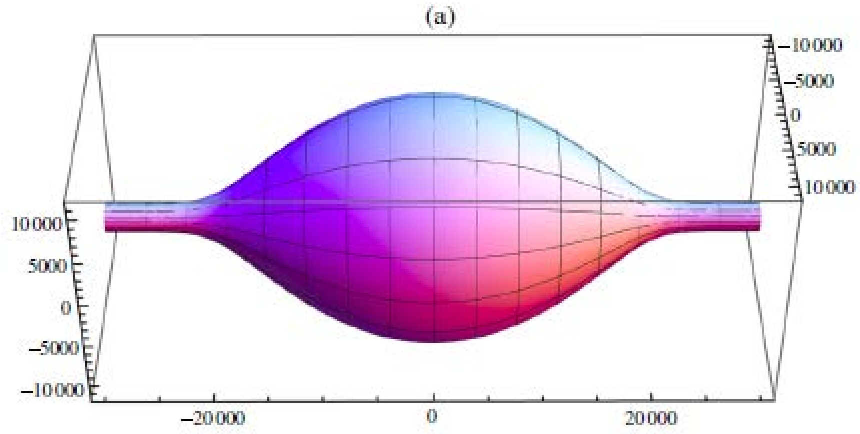
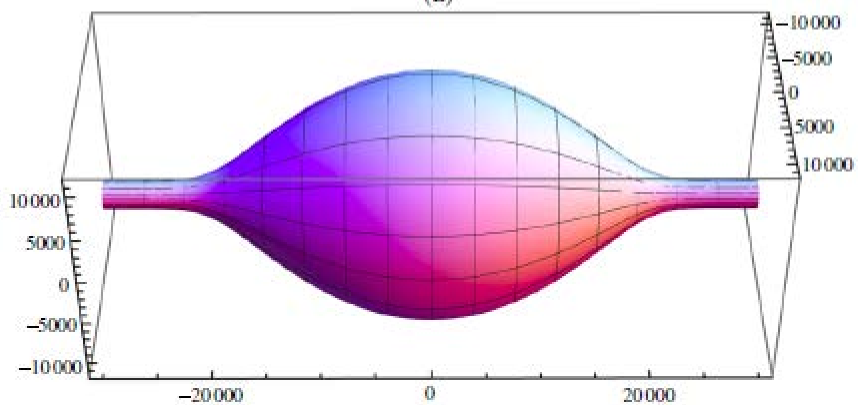


FIG. 2 (color online). The radius as a function of ρ.

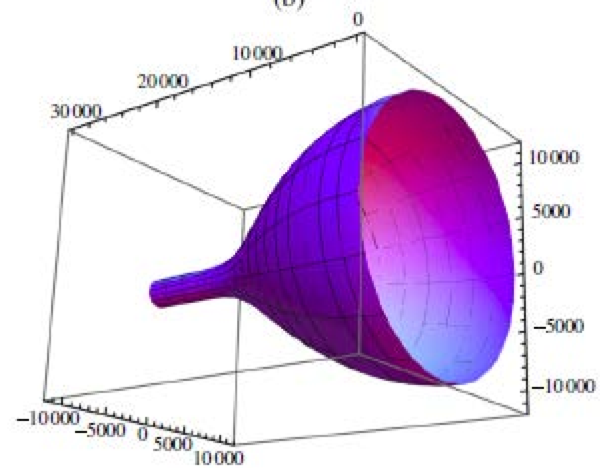


(a)



-10000
-5000
0
5000
10000
10000
5000
0
-5000
-10000
-20000
0
20000

(b)



0
10000
20000
30000
10000
5000
0
-5000
-10000
-10000
-5000
0
5000
10000

5. Summary and Discussions

- We reviewed the formulation of the bubble.
- False vacua exist e.g., in non-minimally coupled theory.
- Vacuum bubbles with finite geometry, with the radius & nucleation rate
- New Type of the solutions :
  Ex) bubble with compact geometry,
  degenerate vacua in dS, flat, & AdS.
  Oscillating solutions; can make the thick domain wall.
- Similar analysis for the Fubini instanton
- Physical role and interpretation of many solutions are still not clear.
- The application to the braneworld cosmology has been discussed for the model of magnetically charged BH pairs separated by a domain wall in the 4 or 5-dim. spacetime with a cosmological constant.
- Can there be alternative model for the accelerating expanding universe?

Thank you!