QCD under a strong magnetic field

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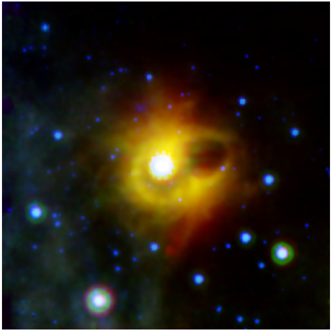
(Based on DKH 98, 2011, 2014)

Motivations

- Magnetic field is relevant in QCD if strong enough:

$$|eB| \gtrsim \Lambda_{\rm QCD}^2 \approx 10^{19}\,{\rm Gauss}\cdot e.$$

- Some neutron stars, called magnetars, have magnetic fields at the surface, $B \sim 10^{12-15}$ G (Magnetar SGR 1900+14):



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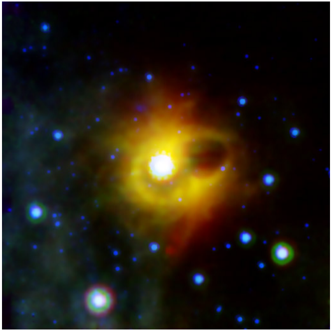

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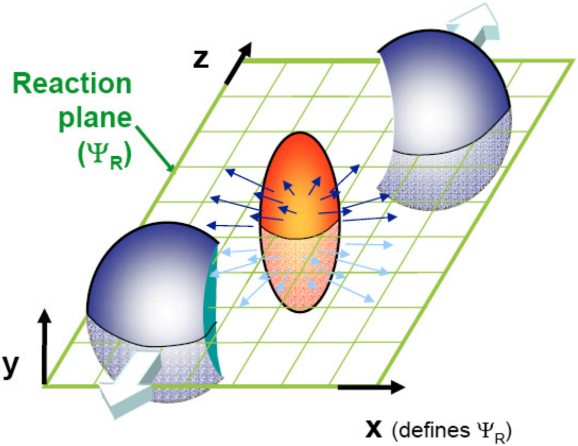
- In the peripheral collisions of relativistic heavy ions huge magnetic fields are produced at the center:



Motivations

Contents

- Vector Condensation (DKH, work under progress)
- Neutron star cooling (DKH, 1998)
- Chiral Magnetic Effect (DKH, 2011)

Vector condensation

- The energy spectrum of (elementary) charged particle under the magnetic field ($\vec{B} = B\hat{z}$):

$$E(\vec{p}) = \pm\sqrt{p_z^2 + m^2 + n|qB|},$$

where $n = 2n_r + |m_L| + 1 - \mathrm{sign}(qB)\,(m_L + 2s_z)$.

- At the lowest Landau level the spin of the rho meson is along the B field direction and $n = -1$. If elementary,

$$m_\rho^2(B) = m_\rho^2 - |eB|\,.$$

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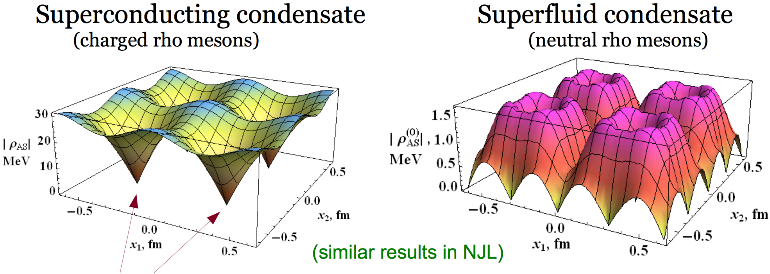
Vector condensation

- Vector meson condensation: Vector order parameter develops under strong magnetic field (Chernodub 2011):

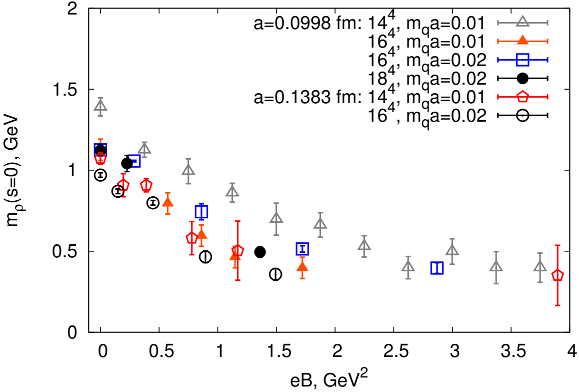
$$\langle \bar{u}\gamma_1 d\rangle = -i\,\langle \bar{u}\gamma_2 d\rangle = \rho(x_\perp)\,.$$



(similar results in NJL)

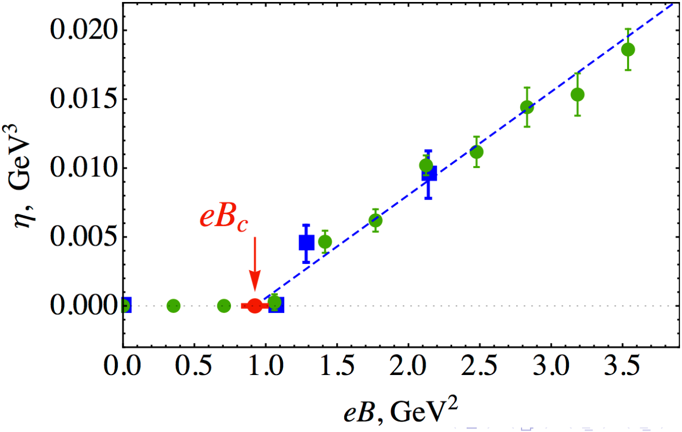
Vector condensation

- Lattice calculation shows vector meson becomes lighter under the B field (Luschevskaya and Larina 2012):



Vector condensation

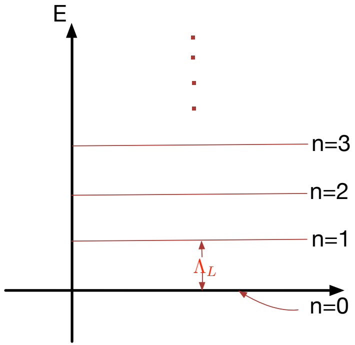
- Lattice calculation shows vector meson condensation at $B > B_c = 0.93\mathrm{GeV}^2/e$ (Barguta et al 1104.3767):



Effective Lagrangian (DKH98 & 2014):

- Quarks under strong B field occupy Landau levels:

$$E = \pm\sqrt{p_z^2 + m^2 + 2n|qB|}, \quad (n = 0, 1, \cdots)$$



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- Quark propagator under B field is given as

$$S_F(x) = \sum_{n=0}^{\infty}(-1)^n \int_k e^{-ik\cdot x} e^{-k_\perp^2/|qB|} S_n(qB,k)$$

$$S_n(qB,k) = \frac{D_n(qB,k)}{[(1+i\epsilon)k_0]^2 - k_z^2 - 2|qB|n}$$

$$D_n = 2\tilde{\not{k}}_{\parallel}\left[P_- L_n\left(\frac{2k_\perp^2}{|qB|}\right) - P_+ L_{n-1}\left(\frac{2k_\perp^2}{|qB|}\right)\right] + 4\not{k}_\perp L^1_{n-1}\left(\frac{2k_\perp^2}{|qB|}\right).$$

Matching with QCD at Λ_L:

- At low energy $E < \Lambda_L$ we integrate out the modes in the higher Landau levels ($n \neq 0$).
- A new quark-gluon coupling:

$$\mathcal{L}_2 = c_2 \frac{i g_s^2}{|qB|} \bar{Q}_0 \not{A} \tilde{\gamma}_\mu \cdot \partial_{\shortparallel} \not{A} \tilde{\gamma}_\mu Q_0 \,.$$

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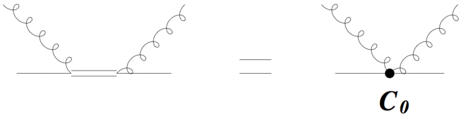

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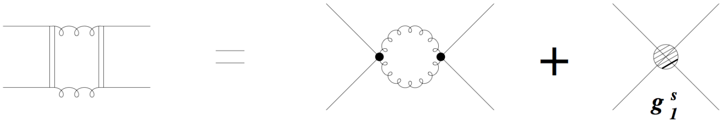
- Four-Fermi couplings for LLL quarks:



$$\mathcal{L}_{\rm eff}^1 \ni \frac{g_1^s}{4\,|qB|}\left[\left(\bar{Q}_0 Q_0\right)^2 + \left(\bar{Q}_0 i\gamma_5 Q_0\right)^2\right] .$$

- Below Λ_L the quark-loop does not contribute to the beta-function of α_s: At one-loop

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\Lambda_L)} + \frac{11}{2\pi}\ln\left(\frac{\mu}{\Lambda_L}\right) .$$

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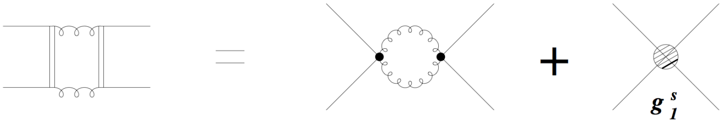

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- One-loop RGE for the four-quark interaction:

$$\mu \frac{d}{d\mu} g_1^s = -\frac{40}{9} \alpha_s^2 \, (\ln 2)^2$$

- Solving RGE to get

$$g_1^s(\mu) = 1.1424 \, (\alpha_s(\mu) - \alpha_s(\Lambda_L)) + g_1^s(\Lambda_L) \, .$$

- If $B \geq 10^{20}\,\mathrm{G}$, the four-quark interaction is stronger than gluon interaction. Therefore the chiral symmetry should break at a scale higher than the confinement scale for $B \geq 10^{20}\,\mathrm{G}$.

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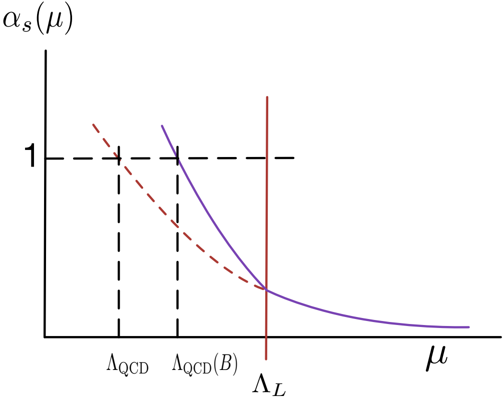
Vector mesons in the Effective Lagrangian:

- Running coupling under strong B field:



Vector mesons in the Effective Lagrangian:

- We need a stronger B field ($B > m_\rho^2/e$) to condense vector mesons:

$$m_\rho^{\mathrm{eff}\,2}(B) = m_\rho^2 \cdot \left(\frac{\Lambda_{\mathrm{QCD}}(B)}{\Lambda_{\mathrm{QCD}}}\right)^2 - |eB|\,.$$

- The critical B field occurs at (DKH 2014)

$$eB_c = m_\rho^2 \cdot \left(\frac{m_\rho}{\Lambda_{\mathrm{QCD}}}\right)^{\frac{4}{9}} \approx 0.90\ \mathrm{GeV}^2\,.$$

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QCD Vacuum Energy:

- The additional vacuum energy at one-loop is given by Schwinger as

$$\Delta\mathcal{E}_{\rm vac} = -\frac{1}{16\pi^2}\int_0^\infty \frac{ds}{s^3}e^{-M_\pi^2 s}\left[\frac{eBs}{\sinh(eBs)} - 1\right].$$

- The chiral condensate becomes by the Gell-Mann-Oakes-Renner relation (Shushpanov+Smilga '97)

$$\langle\bar{q}q\rangle^B = \langle\bar{q}q\rangle^{B=0}\left(1 + \frac{|eB|\ln 2}{16\pi^2 F_\pi^2} + \cdots\right).$$

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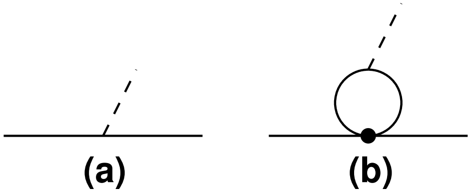

- Energy loss per unit volume per unit time at low temp. (Iwamoto, Ellis, Brinkman+Turner)

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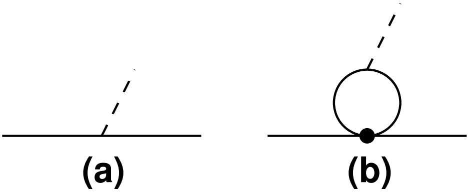

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- The axion coupling $g_{an} \propto m_n/f_{PQ}$, $m_n \propto \langle \bar{q}q \rangle$, and since f_{PQ} does not get any corrections from the magnetic field, we have

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- The correction then becomes (DKH '98), using the Goldberg-Treiman relation and GOR relation,

$$\frac{\mathcal{Q}_a^1(B)}{\mathcal{Q}_a^1(0)} \simeq \left(1 + \frac{g_1^s}{2\pi^2}\right)^2 \left(1 + \frac{|eB| \ln 2}{16\pi^2 F_\pi^2}\right)^{6.5}.$$

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Chiral Magnetic Effects (DKH2011)

- An instanton number may be created in RHIC:

$$n_w = \frac{g_s^2}{32\pi^2} \int \mathrm{d}^4 x \, G^a \tilde{G}^a = n_L - n_R \, .$$

whose conjugate variable is μ_A.

- Chiral magnetic effect (Fukushima+Kharzeev+Warringa): Under a strong magnetic field

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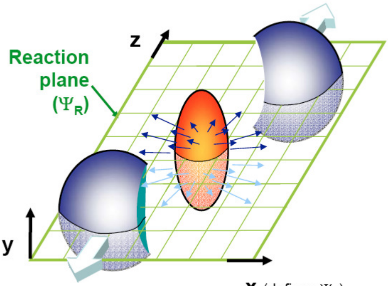

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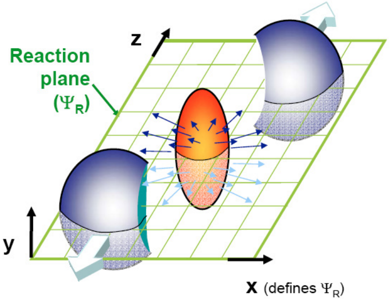

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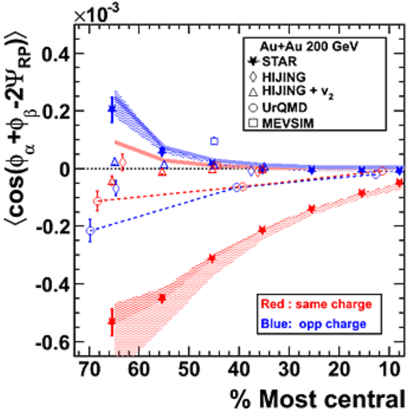

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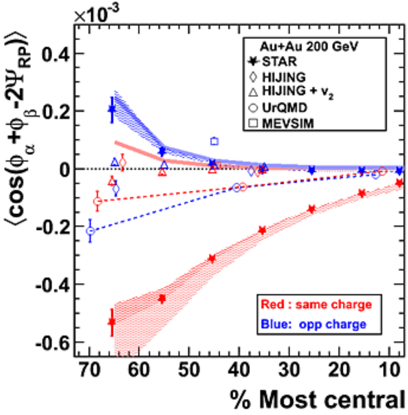

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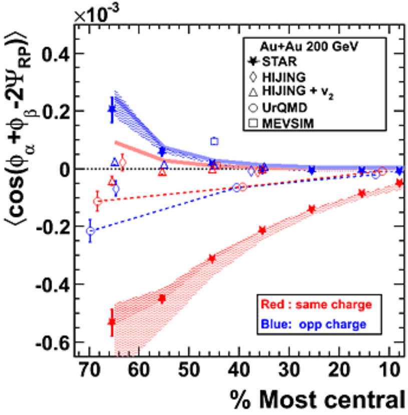

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$$\Delta^\alpha_L(\mu_L) \equiv \langle \bar{\psi}_L \gamma^\alpha \psi_L \rangle = -\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{Tr}\left(\gamma^\alpha \tilde{S}(k)_L \right) .$$

- Matter-dependent part is finite and explicitly calculable:

$$\Delta^\alpha_{\mathrm{mat}} \equiv \Delta^\alpha(\mu_L, B) - \Delta^\alpha(0, B) = \int_0^{\mu_L} \mathrm{d}\mu' \frac{\partial}{\partial \mu'} \Delta^\alpha(\mu', B)$$

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- Integrating over $\vec{k}_\perp$, we get

$$\Delta^\alpha_{\rm mat} = |qB|\left[\Gamma_L^{\alpha\beta} I_\beta^{(0)} + 2g_{\shortparallel}^{\alpha\beta}\sum_{n=1} I_\beta^{(n)}\right],$$

where $\Gamma_L^{\alpha\beta} = \epsilon^{\alpha\beta 12}\,{\rm sign}(qB) + g_{\shortparallel}^{\alpha\beta}$ and

$$I^{(n)\beta} = \int_0^\mu {\rm d}\mu' \int_{k_{\shortparallel}} k_{\shortparallel}^\beta\,\delta\left(k_{\shortparallel}^2 - \Lambda_n\right)\cdot\delta(k^0-\mu') = \frac{p_F^{(n)}}{4\pi^2}\delta^{\beta 0}.$$

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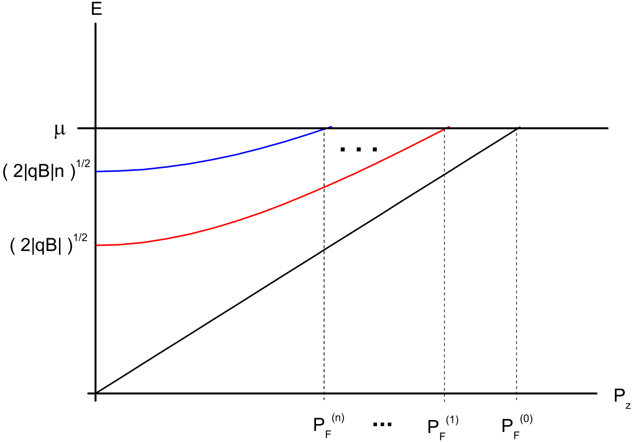
$$I^{(n)\beta} = \int_0^\mu {\rm d}\mu' \int_{k_\shortparallel} k_\shortparallel^\beta \delta\left(k_\shortparallel^2 - \Lambda_n\right)\cdot\delta(k^0-\mu') = \frac{p_F^{(n)}}{4\pi^2}\delta^{\beta 0}\,.$$

The Fermi momentum at the n-th Landau level:

$$p_F^{(n)}(\mu, B) = \begin{cases} \sqrt{\mu^2 - 2|qB|n}, & \text{if } \mu > 2|qB|n; \\ 0, & \text{otherwise.} \end{cases}$$



- The density of states

$$n_L = \frac{|qB|}{4\pi} \cdot \sum_n \frac{p_F^{(n)}(\mu_L, B)}{\pi} \,.$$

- As $\Delta^\alpha(0, B) = 0$, the anomalous electric (axial) vector currents become

$$J_V^\alpha \equiv q\,(\Delta_L^\alpha + \Delta_R^\alpha) \;=\; \delta^{\alpha 3}\,\frac{q^2 B}{2\pi^2}\mu_A + \delta^{\alpha 0}\,q\,n\,,$$

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- Full contributions to the anomalous current:

$$\langle J^{\alpha} \rangle = \frac{\delta \Gamma_{\mathrm{mat}}(A, G\,; \mu)}{\delta A_{\alpha}}\bigg|_{A=0=G} .$$

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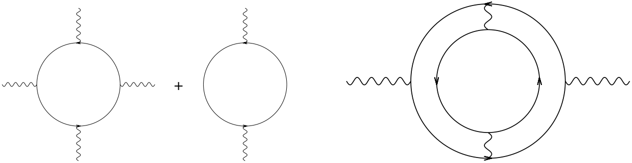

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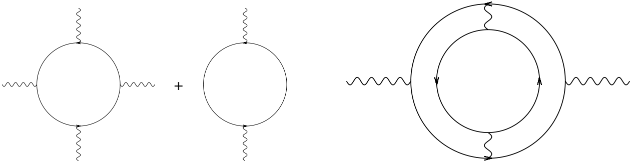

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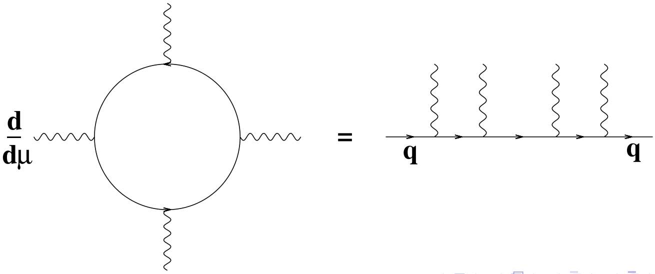

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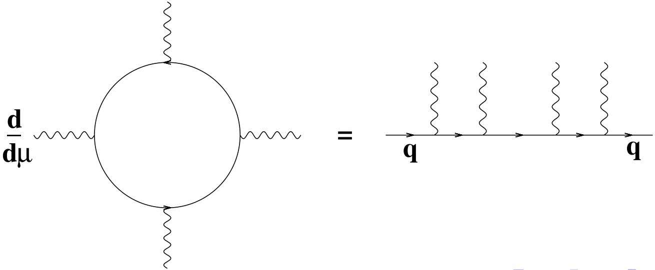

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- Magnetic field is relevant in QCD if $B \geq 10^{19}\,\mathrm{G}$.
- We derive an effective theory for LLL QCD, which has a new marginal four-quark interactions.
- Scale separation between chiral symmetry breaking and confinement.
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- Condensation along the vector channel occurs, when

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- We show that magnetars cool too quickly by emitting axions if $B > 10^{20}\,\mathrm{G}$.
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