Massive Photon and Cosmology

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$$m_\gamma < 10^{-32}\mathrm{eV}$$

Contents

Massive Photon: Introduction

♦ Maxwell-Proca Equations

♦ de Broglie: Photon mass would lead to a larger speed of violet light than that of the red one

♦ Schrödinger: Magnetic field of the Earth would be exponentially cut off at distances of the order of the photon Compton wave length $\lambda_\gamma \sim 1/m_\gamma$

♦ The longitudinal photons do not manifest themselves in the black body radiation: Bass and Schrödinger

Massive Photon: Introduction

♦ Coester, Umezawa, Glauber, Stueckelberg, Boulware and Gilbert: massive QED smoothly goes over to QED. Amplitudes describing longitudinal modes suppressed as $m_\gamma \to 0$, provided that electromagnetic current is conserved

♦ Deser: Longitudinal mode is transformed into a scalar mode decoupled from the current, remains coupled to gravitation

♦ Reviews on massive photon: Goldhaber and Nieto (2010)

♦ Reviews on Stuckelberg theory: Ruegg and Ruiz-Altaba (2003)

Massive QED and Cosmology

Invariant Action

$$\mathcal{L}_{inv} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2(A_\mu + \frac{1}{m}\partial_\mu\phi)(A^\mu + \frac{1}{m}\partial^\mu\phi)$$

$$+\bar{\psi}(i\gamma^\mu\partial_\mu - m_f)\psi + A_\mu\bar{\psi}\gamma^\mu\psi$$

- Invariant under $A_\mu \rightarrow A_\mu - \partial_\mu\Lambda,\ \phi \rightarrow \phi + m\Lambda,\ \psi \rightarrow e^{-i\Lambda}\psi$

Massive QED and Cosmology

Invariant Action

$$\mathcal{L}_{inv} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2(A_\mu + \frac{1}{m}\partial_\mu\phi)(A^\mu + \frac{1}{m}\partial^\mu\phi)$$

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- Invariant under $A_\mu \to A_\mu - \partial_\mu\Lambda,\ \phi \to \phi + m\Lambda,\ \psi \to e^{-i\Lambda}\psi$

Gauge fixed action

$$\mathcal{L}_\xi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2A_\mu A^\mu \ - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\xi m^2\phi^2$$

$$-\frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m_f)\psi + A_\mu\bar{\psi}\gamma^\mu\psi$$

Massive QED and Cosmology

- Propagator:

$$A: \quad \left[g^{\mu\nu} - \frac{(1-\xi)k^\mu k^\nu}{k^2+\xi m^2}\right]\frac{-1}{k^2+m^2}$$

$$\phi: \quad \frac{-1}{k^2-\xi m^2}$$

- $m \to 0$ exists, gives the gauge-fixed massless propagator
- Good high energy behavior, provided $\xi \neq \infty$

$$\frac{-1}{k^2+m^2}\left[g^{\mu\nu} + \frac{k^\mu k^\nu}{m^2}\right] + \frac{k^\mu k^\nu}{m^2(k^2+\xi m^2)}$$

- $\xi \to \infty$ gives Proca propagator
- ξ dependent poles cancel each other: In the limit, $\xi \to \infty$, they disappear

Massive QED and Cosmology

- Unitary and Renormalizable
- Dirac Monopole
- Infrared cutoff
- Indistinguishable from QED with photon mass limit

Massive QED and Cosmology

(arXiv:1305.4438)

$$S = S' + S_{gf}$$

$$S' = \int d^4x \, \sqrt{-g}\left[\frac{R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu\right] + \int d^4x \, \sqrt{-g} \, \mathcal{L}_D \, ,$$

$$\mathcal{L}_D = -\frac{i}{2}\left[(\nabla_\mu \bar{\psi})\gamma^\mu \psi - \bar{\psi}\gamma^\mu \nabla_\mu \psi\right] - m_f \bar{\psi}\psi + A_\mu \bar{\psi}\gamma^\mu \psi.$$

$$S_{gf} = -\frac{1}{2\alpha}\int d^4x \sqrt{-g}\left[\left(\overset{\{\}}{\nabla}_\mu \, A^\mu\right)^2 + 4\xi S_\mu A^\mu \, \overset{\{\}}{\nabla}_\nu \, A^\nu + 4\xi^2 \left(S_\mu A^\mu\right)^2\right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \Gamma^\rho_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} - K_{\mu\nu}{}^\rho$$

$$K_{\mu\nu}{}^\rho = -\left(S_{\mu\nu}{}^\rho + S^\rho{}_{\mu\nu} + S^\rho{}_{\nu\mu}\right)$$

Massive QED and Cosmology

$$G_{\mu\nu}(\{\}) = T_{\mu\nu}(T, A) + T_{\mu\nu}(D)\,,$$

$$\begin{aligned} T_{\mu\nu}(T,A) \;=\; & \frac{8}{3}\left[S_\mu S_\nu - \frac{1}{2}g_{\mu\nu}S_\alpha S^\alpha\right] + \left[\tilde{K}_{\alpha(\mu}{}^{\beta}\tilde{K}_{\nu)\beta}{}^{\alpha} - \frac{1}{2}g_{\mu\nu}\tilde{K}_{\alpha\beta\gamma}\tilde{K}^{\alpha\beta\gamma}\right] \\ & + \left[F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right] + m^2\left[A_\mu A_\nu - \frac{1}{2}g_{\mu\nu}A_\alpha A^\alpha\right] \\ & + \frac{g_{\mu\nu}}{\alpha}\left[\frac{1}{2}\left(\overset{\{\}}{\nabla}_\mu A^\mu\right)^2 + A^\nu \overset{\{\}}{\nabla}_\nu \left(\overset{\{\}}{\nabla}_\alpha A^\alpha + 2\xi S_\alpha A^\alpha\right) - 2\xi^2\left(S_\alpha A^\alpha\right)^2\right] \\ & + \frac{1}{\alpha}\left[-2A_{(\mu}\overset{\{\}}{\nabla}_{\nu)}\left(\overset{\{\}}{\nabla}_\alpha A^\alpha + 2\xi A_\alpha S^\alpha\right) + 4\xi S_{(\mu}A_{\nu)}\overset{\{\}}{\nabla}_\alpha A^\alpha + 8\xi^2 S_{(\mu}A_{\nu)}\left(S_\alpha A^\alpha\right)\right] \end{aligned}$$

$$T_{\mu\nu}(D) = \frac{i}{4}\left[(D_\mu\bar{\psi})\gamma_\nu\psi + (D_\nu\bar{\psi})\gamma_\mu\psi - \bar{\psi}\gamma_\mu D_\nu\psi - \bar{\psi}\gamma_\nu D_\mu\psi\right] + g_{\mu\nu}\mathcal{L}_D(\{\})$$

$$0 = \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right) + m^2A^\mu - \frac{1}{\alpha}\left[\overset{\{\}}{\nabla}{}^\mu\left(\overset{\{\}}{\nabla}_\nu A^\nu\right) - 2\xi S^\mu\overset{\{\}}{\nabla}_\nu A^\nu + 2\xi\overset{\{\}}{\nabla}{}^\mu\left(S_\nu A^\nu\right) - 4\xi^2 S^\mu S_\nu A^\nu\right] - \bar{\psi}\gamma^\mu\psi$$

$$S^\mu = -\frac{3\xi}{4\alpha}\left[A^\mu\overset{\{\}}{\nabla}_\nu A^\nu + 2\xi A^\mu S_\nu A^\nu\right] \qquad K_{[\mu\nu\rho]} = \frac{i}{4}\bar{\psi}\gamma^{[\rho}\gamma^\nu\gamma^{\mu]}\psi$$

$$\left(i\gamma^\mu\overset{\{\}}{\nabla}_\mu - m_f + \gamma^\mu A_\mu\right)\psi = -\frac{i}{4}\gamma^{[\rho}\gamma^\nu\gamma^{\mu]}\psi K_{\mu\nu\rho}$$

Massive QED and Cosmology

$$ds^2 = -dt^2 + a^2(t)dx^i dx^i$$

$$S_{10}{}^{1} = S_{20}{}^{2} = S_{30}{}^{3} = h(t)/2$$

$$A_\mu = (f(t), 0, 0, 0)$$

$$\bar{\psi}\psi = \frac{A}{a^3}, \quad \psi^\dagger\psi = \frac{B}{a^3}$$

$A = B(> 0)$ for spinor configuration $\psi = (\psi_1, 0, 0, 0)$

$A = -B(< 0)$ for $\psi = (0, 0, 0, \psi_4)$

Massive QED and Cosmology

asymptotic de Sitter phase

$$\sqrt{\Lambda(t)} \equiv \sqrt{\frac{1}{2|\alpha|}\left[\dot{f} + 3\,(H + \xi h)\, f\right]}$$

$$\boxed{\Lambda = \frac{|\alpha|}{3\xi^2} m^2} \qquad 3h_*^2 = \frac{m^2}{2} f_*^2$$

$$h_*^2(\xi) = \frac{|\alpha| m^2}{18\xi^4}\left[1 - 2\xi^2 \pm \sqrt{1 - 4\xi^2}\right], \quad f_*(\xi) = \frac{\sqrt{|\alpha|}}{\sqrt{6}\,\xi^2}\left(1 \pm \sqrt{1 - 4\xi^2}\right)$$

Massive QED and Cosmology

asymptotic expansions

$$f = f_*(\xi)\left(1 + \frac{f^{(3)}}{a^3} + \frac{f^{(6)}}{a^6} + \cdots\right), \quad h = h_*(\xi)\left(1 + \frac{h^{(3)}}{a^3} + \frac{h^{(6)}}{a^6} + \cdots\right)$$

$$3H^2 \simeq \frac{|\alpha| m^2}{3\xi^2} + \frac{\left(m_f + \frac{4|\alpha| f_*}{-2|\alpha|+3\xi^2 f_*^2}\right) B}{a^3} + \frac{\left(\frac{2|\alpha|+3\xi^2 f_*^2}{-2|\alpha|+3\xi^2 f_*^2}\right) B f_* f^{(3)}}{a^6},$$

$$-3H^2 - 2\dot{H} \simeq \frac{-|\alpha| m^2}{3\xi^2} + \frac{\left(\frac{2|\alpha|+3\xi^2 f_*^2}{-2|\alpha|+3\xi^2 f_*^2}\right) B f_* f^{(3)}}{a^6}.$$

(i) systematic expansions in powers of B/m^2

(ii) higher order terms starting from $1/a^6$ as a deviation from ΛCDM

(iii) $1/a^6$ term contributes a *positive* energy density

Massive QED and Cosmology

DATA ANALYSIS

$$\frac{H^2(z)}{H_0^2} = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_s(1+z)^6$$

$$\Omega_s \sim 10^{-14}$$

$$B/m^2 \sim 10^{-7} \text{ for } \mathcal{O}(\xi), \mathcal{O}(|\alpha|) \sim 1$$

Summary

♦ Massive QED could be a viable model for dark energy

♦

$$\boxed{\Lambda \sim m_\gamma^2}$$

♦ The vacuum $A_\mu = f_\mu$ breaks Lorentz symmetry

♦ A small deviation from ΛCDM

♦ Non-minimal coupling of the Stuckelberg scalar field to gravity

♦ ξ-(in)dependence in quantum cosmology?

Dark Massive Vector and Cosmology

$$S_{inv} = \int d^4x \Big\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu}$$
$$-\frac{1}{2}m^2(A_\mu + \alpha S_\mu + \frac{1}{m}\partial_\mu\phi_1)(A^\mu + \alpha S^\mu + \frac{1}{m}\partial^\mu\phi_1)$$
$$-\frac{1}{2}\mu^2(S_\mu + \frac{1}{\mu}\partial_\mu\phi_2)(S^\mu + \frac{1}{\mu}\partial_\mu\phi_2)\Big\}$$

- Invariant under $A_\mu \to A_\mu - \partial_\mu\Lambda_1, S_\mu \to S_\mu - \partial_\mu\Lambda_2$
  $\phi_1 \to \phi_1 + m\Lambda_1 + \alpha m\Lambda_2,\ \phi_2 \to \phi_2 + \mu\Lambda_2$

Double Massive Vector and Cosmology

Gauge fixing term:

$-\frac{1}{2\xi_1}\left[\partial_\mu A^\mu + \alpha\partial_\mu S^\mu + \xi_1 m\phi_1 + \lambda(A_\mu + \alpha S_\mu)S^\mu\right]^2$

$-\frac{1}{2\xi_2}(\partial_\mu S^\mu + \xi_2\mu\phi_2)^2$

$$S_{gf} = \int d^4x \Big\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu}$$

$$-\frac{1}{2}m^2(A_\mu + \alpha S_\mu)(A^\mu + \alpha S^\mu) - \frac{1}{2}\mu^2 S_\mu S^\mu$$

$$-\frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\xi_1 m^2\phi_1^2 - \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}\xi_2\mu^2\phi_2^2$$

$$-\frac{1}{2\xi_1}\left[(\partial_\mu A^\mu + \alpha\partial_\mu S^\mu) + \lambda(A_\mu + \alpha S_\mu)S^\mu\right]^2$$

$$-\frac{1}{2\xi_2}(\partial_\mu S^\mu)^2 - \lambda m\phi_1(A_\mu + \alpha S_\mu)S^\mu \Big\}$$

Dark Massive Vector and Cosmology

Redefine: $A_\mu + \alpha S_\mu \to A_\mu,\ \sqrt{1+\alpha^2} S_\mu \to S_\mu,\ \frac{\alpha}{\sqrt{1+\alpha^2}} \equiv \chi$

$$S_{gf} = \int d^4x \Big\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\chi}{2} F_{\mu\nu} C^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu}$$

$$-\frac{1}{2} m^2 A_\mu A^\mu - \frac{1}{2} \mu^2 (1-\chi^2) S_\mu S^\mu$$

$$-\frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{1}{2} \xi_1 m^2 \phi_1^2 - \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - \frac{1}{2} \xi_2 \mu^2 \phi_2^2$$

$$-\frac{1}{2\xi_1} \left[\partial_\mu A^\mu + \lambda \sqrt{1-\chi^2} A_\mu S^\mu \right]^2$$

$$-\frac{1}{2\xi_2} (1-\chi^2) (\partial_\mu S^\mu)^2 - \lambda \sqrt{1-\chi^2} m \phi_1 A_\mu S^\mu \Big\}$$

Dark Massive Vector and Cosmology

$$S = \int d^4x \, \sqrt{-g} \Big\{ \frac{R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\chi}{2} F_{\mu\nu} C^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu - \frac{1}{2} \mu^2 S_\mu S^\mu + \frac{1}{2\alpha} \left[\nabla_\mu A^\mu + \xi \left(S_\mu A^\mu \right) \right]^2 + \frac{1}{2\beta} (\nabla_\mu S^\mu)^2 \Big\}$$

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

Constant solution:

$$A_\mu = f_\mu, \qquad S_\mu = h_\mu$$

Dark Massive Vector and Cosmology

$$ds^2 = -dt^2 + a(t)^2 dx_i dx_i$$

$$T_{\mu\nu} = F_{\mu\alpha} F_{\nu}{}^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - m^2 \left[A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A_\alpha A^\alpha \right]$$

$$+ C_{\mu\alpha} C_{\nu}{}^{\alpha} - \frac{1}{4} g_{\mu\nu} C_{\alpha\beta} C^{\alpha\beta} + \mu^2 \left[S_\mu S_\nu - \frac{1}{2} g_{\mu\nu} S_\alpha S^\alpha \right]$$

$$- \frac{g_{\mu\nu}}{2\alpha} \left[(\nabla_\mu A^\mu) + \xi \left(S_\alpha A^\alpha \right)^2 \right]^2 - \frac{g_{\mu\nu}}{2\beta} \left(\nabla_\mu S^\mu \right)^2$$

$$= -\Lambda g_{\mu\nu}$$

Dark Massive Vector and Cosmology

$$\Lambda = \frac{\alpha}{2\xi^2} m^2 \mu^2 \left[1 - 4\frac{\alpha}{\beta} \left(\frac{m^2}{\mu^2} \right) \left(1 - \frac{\xi^2}{\alpha m^2} h_\mu h^\mu \right) \right]$$

- Proca limit: $\beta \to \infty$

$$\boxed{\Lambda = \frac{\alpha}{2\xi^2} m^2 \mu^2}$$

$$h_\mu h^\mu = \frac{3}{4} \left(\frac{\alpha m^2 \mu^2}{\xi^4} \right) \left[\frac{4\xi^2}{3\mu^2} - 1 \pm \sqrt{1 - \frac{8\xi^2}{3\mu^2}} \right]$$

$$\mu^2 \geq \frac{8}{3}\xi^2$$

$$\Lambda \geq \frac{4}{3}\alpha m^2$$

Summary

- $$\rho_\Lambda \sim m^2\mu^2$$
- Inclusion of Stuckelberg fields for cosmology
- Unified description of dark matter and dark energy
- Cosmic coincidence: $\rho_\Lambda \sim \left(\frac{M_{EW}^2}{M_{pl}}\right)^4$

  (Alkani-Hamed et al, 2000)
- Photon oscillations ?
- EW theory with a massive photon (Ruegg and Ruiz-Altaba, 2003)

Concluding Remarks

- 'Nature abhors massless particles with long-range interactions between them..' (Coleman and Weinberg, 1973)

Concluding Remarks

- 'Nature abhors massless particles with long-range interactions between them..' (Coleman and Weinberg, 1973)

- Photon mass as a constant of nature? What is the principle associated with it?