6d (2,0) Theories and M5 Branes

Kimyeong Lee
KIAS

KIAS
June 2014

Ho-Ung Yee, Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, Eunkyung Ko,
Stefano Bolognesi,

Dongsu Bak, Seok Kim's talk

6d (2,0) Superconformal Theories

- A, D, E type: type IIB on $R^{1+5} \times C^2/\Gamma_{ADE}$
  - A_{N-1}, D_N type on N M5 branes, N M5 (+OM5)
- superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times USp(4)_R$
- fields: B, Φ_I $_{(I=1,2,3,4,5)}$, Ψ : 3 + 5+ 8 d.o.f.
  - selfdual strength H=dB=*H, purely quantum $\hbar=1$
  - Weyl and symplectic-Majorana Ψ
- Nonabelian formulation?.
  - Sorokin, Chu, Ho, Lambert, Papageorgakis, Samtleben et.al, ...
  - Their 4d-reduction implies a local nonabelian field theory which has both electric and magnetic objects.
- N^3 degrees of freedom: Klebanov, Tseytlin, Harvey, Minasina, Moore, P. Yi, Intriligator, Hee-Cheol and Seok Kim, Kallen at.al,

Outline

- 5d Approach: R^{1+4}
  - instantons
  - 1/4 BPS webs
  - Origin of Yang-Mills Coupling
  - Index functions of D
- 6d approach: RxS^5
  - Twisting
  - Z_K-modding
- 5d approach: $RxCP^2$
  - Minkowski and Euclidean Lagrangian
  - Index function
- Conclusion

5-dim Approach

- Douglas, Lambert-Papageorgakis-Schmidt-Sommerfeld
- Nonabelian 6d (2,0) theory of gauge group SU(N) on N M5 branes
- S^1: $x^5 \sim x^5 + 2\pi R$, 5d N=2 U(N) YM Theory (dynamics on D4 branes):
- Instantons= D0 on D4 = Kaluza-Klein modes , $4\pi^2/g^2 = 1/R$
    - threshold bound states of k instantons for Kaluza-Klein momentum k
    - strong coupling limit is 6d (2,0) theory
- perturbative approach: problem at 6-loop (Bern et.al. 1210):
    - incomplete? (Need higher order operators to complete the theory?)

Dyonic Instantons in 5d N=2 SYM

- Index for BPS states with k instantons

$$Q = Q^{\dot{+}}_{+} \qquad \left.\begin{matrix} SU(2)_{2R} \\ SU(2)_{1R} \end{matrix}\right\} \Rightarrow SU(2)_R$$

$$I_k(\mu^i, \gamma_1, \gamma_2, \gamma_3) = \mathrm{Tr}_k \left[(-1)^F e^{-\beta Q^2} e^{-\mu^i \Pi_i} e^{-i\gamma_1 (2J_{1L}) - i\gamma_2 (2J_{2L}) - i\gamma_R (2J_R)} \right]$$

adjoint hyper flavor

  - μ_i : chemical potential for $U(1)^N \subset U(N)_{color}$
  - $\gamma_1, \gamma_2, \gamma_R$: chemical potential for $SU(2)_{1L}$, $SU(2)_{2L}$, $SU(2)_R$

- calculate the index by the localization:

$$I(q, \mu^i, \gamma_{1,2,3}) = \sum_{k=0}^{\infty} q^k I_k$$

- 5d *N=2** instanton partition function on $R^4 \times S^1$: $t \sim t + \beta$

- In $\beta \to 0$ and small chemical potential limit, the index becomes 4d Nekrasov instanton partition function :

$$a_i = \frac{\mu_i}{2} \qquad -\epsilon_1 = i\frac{\gamma_1 - \gamma_R}{2} \qquad \epsilon_2 = i\frac{\gamma_1 + \gamma_R}{2}, \quad m = i\frac{\gamma_2}{2} \qquad q = e^{2\pi i \tau}$$

Scalar Vev — Omega deformation parameter — Adj hypermultiplet mass — instanton fugacity

Monopole & Selfdual String Junction

- 1/16 BPS equations for dyonic monopole string junction
  - $SO(4)_{\text{rotation}} = SO(4)_R \subset SO(5)_R$

$$F_{ab} = \epsilon_{abcd} D_c \phi_d - i[\phi_a, \phi_b], \;\; D_a \phi_a = 0$$

$$F_{a0} = D_a \phi_5, \;\; D_a^2 \phi_5 - [\phi_a, [\phi_a, \phi_5]] = 0$$

- 1/4 BPS equations for monopole string junction

$$F_{12} = D_3 \phi_4 - D_4 \phi_3, F_{23} = D_1 \phi_4, F_{31} = D_2 \phi_4$$

$$F_{41} = D_2 \phi_3, F_{24} = D_1 \phi_3, F_{43} = -i[\phi_4, \phi_3]$$

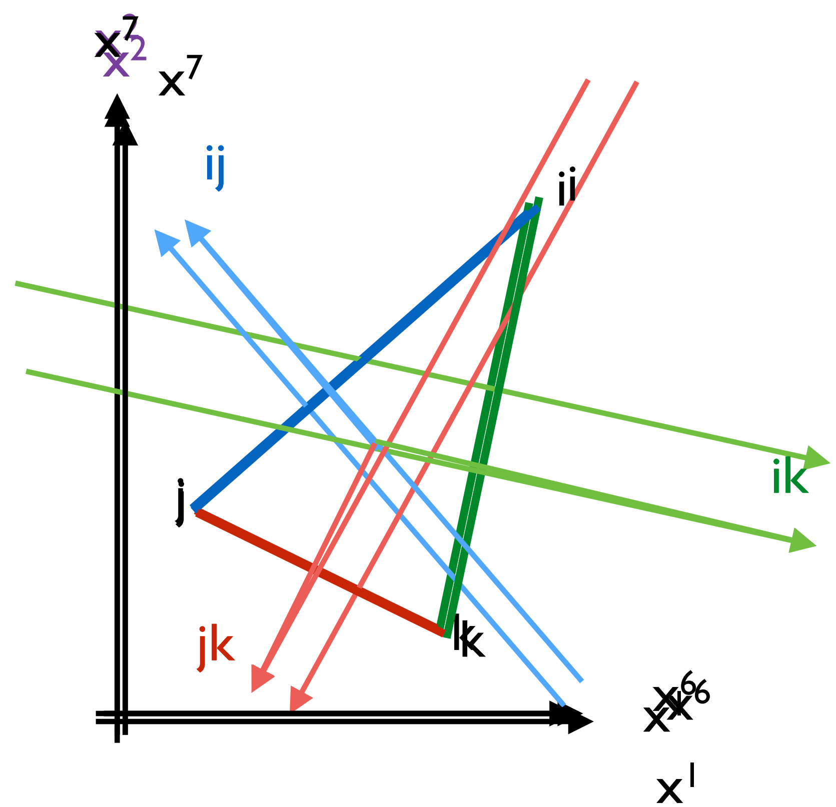
$$D_3 \phi_3 + D_4 \phi_4$$

- 1/16 BPS equations for selfdual strings
  - $SO(5)_{\text{rotation}} = SO(5)_{R:}$ 11-equations

$$H_{0ab} = \partial_a \phi_b - \partial_b \phi_a = \frac{1}{6} \epsilon_{abcde} H_{cde}, \partial_a \phi_a = 0$$

1/4 BPS String Junctions

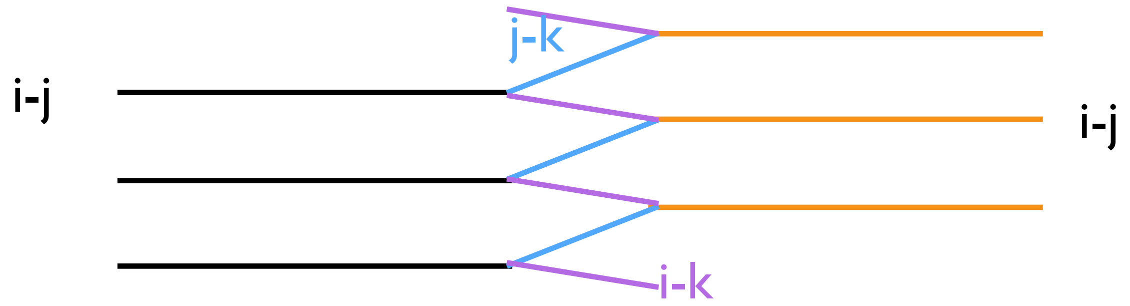
Lock SO(2)rotation on M5 or D4 brane = $SO(2)_R \subset SO(5)_R$



i,j,k M5

6d-5d correspondence in Coulomb phase

- massless abelian tensor H (3), Ψ (8), Φ (5) for i -th M5 brane
  - no wrap: F (3), Ψ (8), Φ (5) on i-th D4 brane
  - wrap: instantons or D0 branes
- selfdual string connecting i-th and j-th M5 branes
  - wrap the circle: W-boson connecting i-th and j-th D4 branes
  - no wrap: monopole strings connecting i-th and j-th D4 branes
- selfdual string junctions connecting i-th, j-th and k-th M5 branes
  - no wrap: monopole string junctions
  - warp:



Counting 1/4 BPS objects in SU(N)

- Selfdual strings with wave on it + junctions
- selfdual strings: N(N-1)/2
- junctions: N(N-1)(N-2)/6
- sum= N(N-1)(N+1)/6=N(N^2-1)/6

$$\rho = \frac{1}{2}\sum_{\alpha>0}\alpha,\ \ \alpha^2 = 2$$

- Weyl vector for ADE

$$2\rho^2 = \frac{1}{2}\sum_{\alpha>0}\alpha^2 + \frac{1}{2}\sum_{\alpha>0\neq\beta>0}\alpha\cdot\beta$$

$$\frac{1}{2}\sum_{\alpha>0}\alpha^2 = \text{number of positive roots}$$

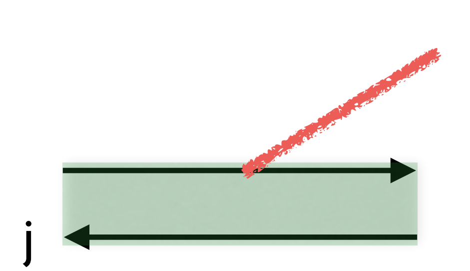
$$\frac{1}{2}\sum_{\alpha>0\neq\beta>0}\alpha\cdot\beta = \text{number of junctions}$$

$$2\rho^2 = \frac{1}{6}(\text{dual Coxeter number}) \times (\text{dimension of group})$$

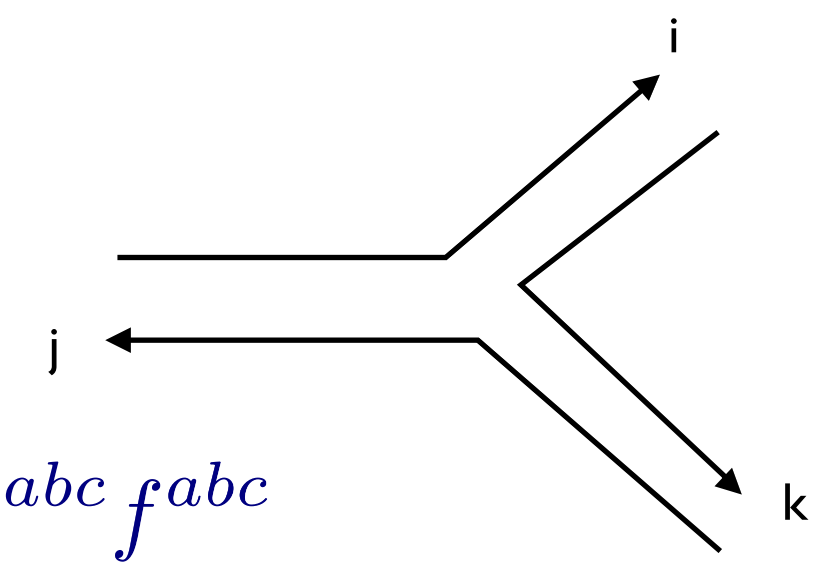
- Anomaly coefficient: 2ρ^2

6d Origin of 5d Yang-Mills Coupling

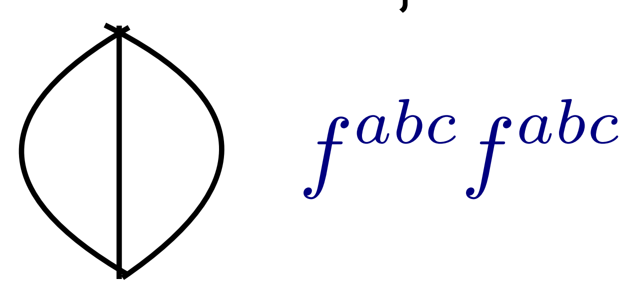
- Off-shell interaction vertex = wrapped junctions
- The structure constant f^{abc}: $[t^a,t^b]=if^{abc} f^c$, $tr_f\, t^a t^b=\delta^{ab}/2$
- $f^{abc}f^{abc} = Tr_{adj}\ T^a T^a = N(N^2-1) = 12\ \rho^2$



- $[E_\alpha, E_{-\alpha}] = \alpha H$, $[E_\alpha,E_\beta] = N_{\alpha\beta}{}^\gamma E_\gamma$
- $f^{\alpha\beta i}$: root- antiroot-cartan: off-shell, wrap the selfdual string around the circle



- $f^{\alpha\beta\gamma}$: root-root-root; off-shell, wrap the string junction world line around the circle.



Kim &Kim: 2-loop effect on S^5 SYM

Effective Lagrangian in Coulomb Phase

- 4d DBI action & 4d YM calculation

$$S_4 = \int d^4x \Big(-\frac{1}{g_4^2}(\partial\phi_4)^2 + \frac{(\partial\phi_4)^4}{\phi_4^4} + \cdots \Big), \quad g_4^2 = \frac{R_5}{R_4}, \quad \phi_4 = \phi_5 = R_5\phi_6$$

- Add just electric KK modes: $\frac{1}{\phi_4^4} \Rightarrow \sum_{n=-\infty}^{\infty} \frac{1}{(\phi_4^2 + n/R_4^2)^2} \sim \frac{R_4}{\phi_4^3}$

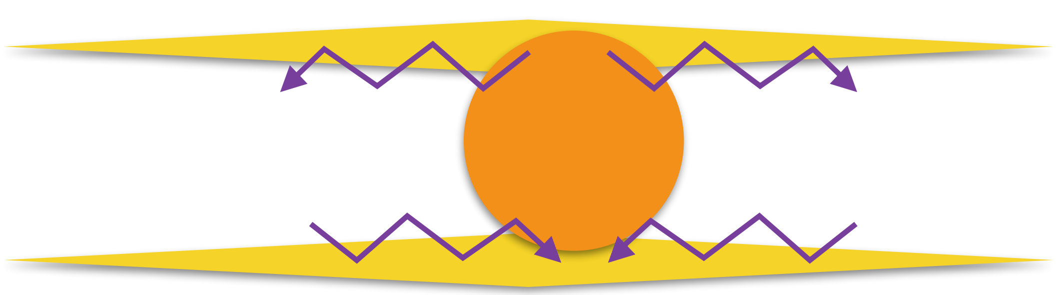
- The 5d effective action

$$S_5 = \int d^4x R_4 \Big(-\frac{1}{g_5^2}(\partial\phi_5)^2 + \frac{(\partial\phi_5)^4}{\phi_5^3} + \cdots \Big), \quad g_5^2 = R_5, \quad \phi_4 = \phi_5 = R_5\phi_6$$

- The 6d effective action

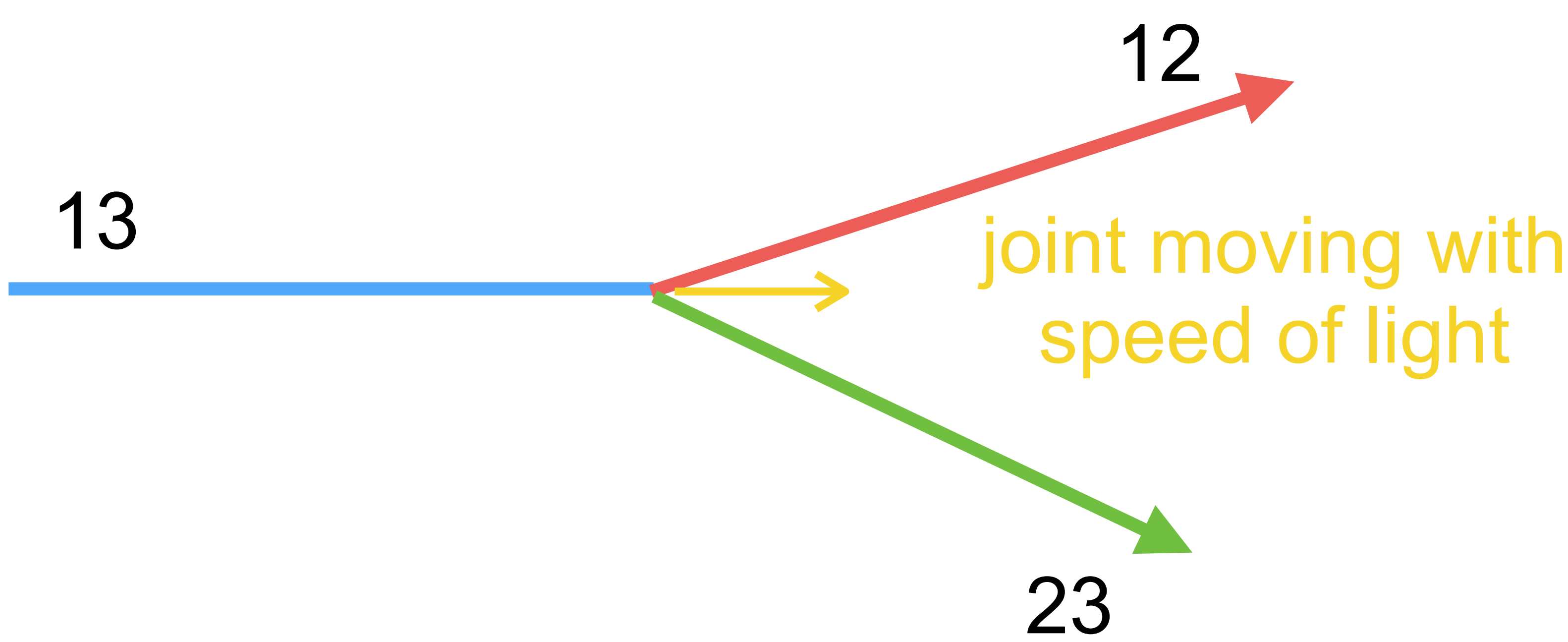
$$S_6 = \int d^4x R_4 R_5 \Big(-(\partial\phi_6)^2 + \frac{(\partial\phi_6)^4}{\phi_6^3} + \cdots \Big),$$

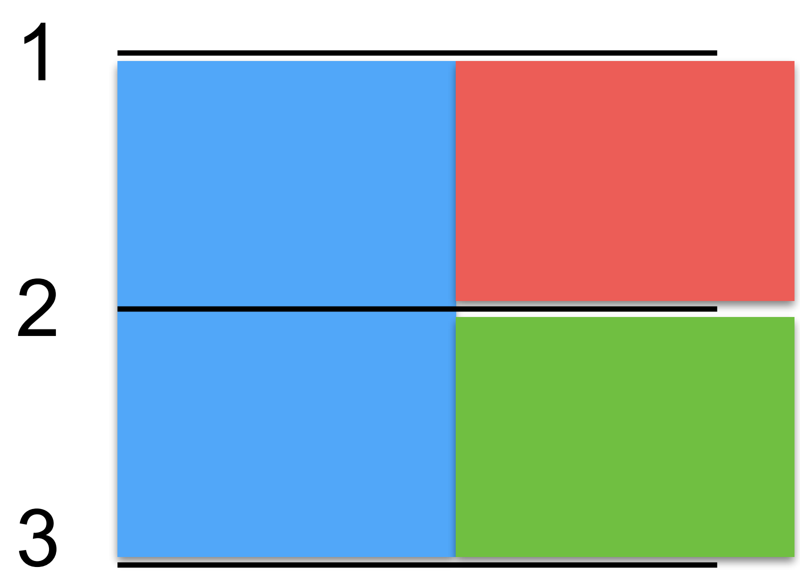
- This matches exactly what J. Schwarz got in 2013 via DBI on gravity background.
- S-dual invariant with only electric KK modes.
- The quantum bubble in 6d is understood.



A striking aspect

- Additional States appearing in dyonic instanton calculations
- Can be regarded as the degenerate limit of junctions
- There is a threshold bound states of W-boson for 13 and instantons on 2nd M5 branes

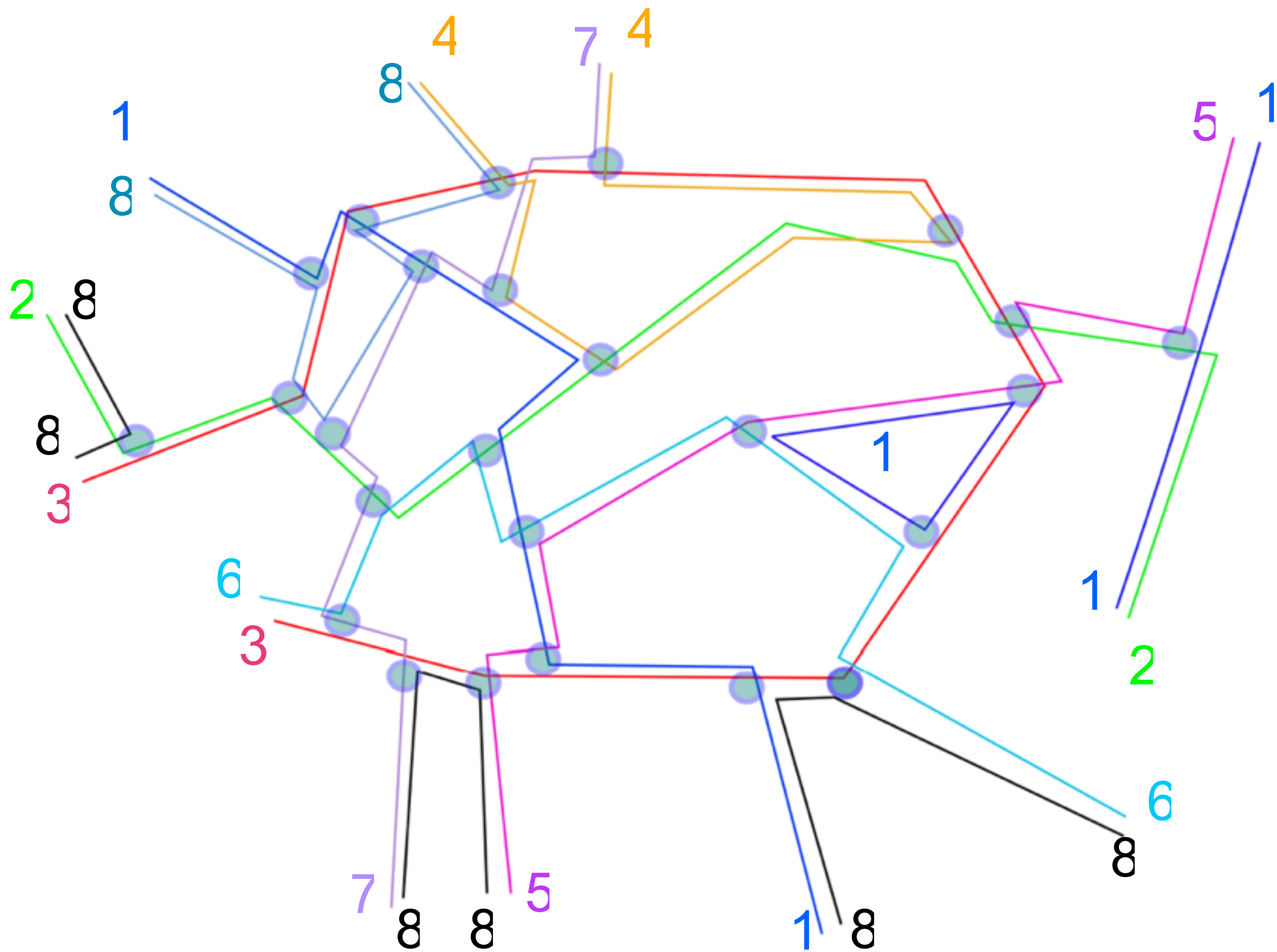




High Temperature in Coulomb Phase

* Micro-canonical
* Massless on N M5 branes: O(N)
* Loops of self-dual strings excitations: $O(N^2)$
* Beyond the Hagedorn temperature
* Webs of junctions and anti-junctions: $O(N^3)$
* Higher order of junctions can be decomposed to elementary junctions
* Excitations of webs of tensionless strings with junctions act as atoms.
    * N^3 degrees of freedom
* Black hole, anomaly, vacuum energy on S^5

Nonzero Temperature in Symmetric Phase



6d (2,0) Theories

- Difficulties with nonabelianization of the B field and its strength H=dB
- Generalize ABJM to M5 brane theory
  - Mode out by R^8/Z_K
  - Weak coupling limit
  - No fixed point
- R^{1+5}/Z_K has a fixed point
- Consider the 6d (2,0) theory on $RxS^{5:}$ TheRadial Quantization
- S^5 = a circle fibration over CP^2
  - $ds^2_{S5} = ds^2_{CP^2} + (dy + V)^2$, $dV = 2J$, $J = -{*}J$, $y \sim y + 2\pi$

- AdS_7 x S^4/Z_K (Tomasiello, 2013): 6d theory with D6 and $\underline{D6}$: still 6d theory with H=*H

Index Function on $S^1 \times S^5$

- Supercharge $$Q^{R_1,R_2}_{j_1,j_2,j_3} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}},\, S = Q^\dagger$$
- BPS bound: $$E = j_1 + j_2 + j_3 + 2(R_1 + R_2)$$
- 6-dim index function:

$$I = \mathrm{Tr}\left[(-1)^F e^{-\beta'\{Q,S\}} e^{-\beta\left(E - \frac{R_1+R_2}{2} - m(R_1-R_2) + aj_1 + bj_2 + cj_3\right)}\right],\ a+b+c=0$$

- Euclidean Path Integral of (2,0) Theory on S^1x S^5
  - S^5= S^1 fiber over CP^2 : -i ∂_y = KK modes

$$k \equiv j_1 + j_2 + j_3$$

- Z_K modding keeps only k/K=integer modes

6d Abelian Theory (Fermion+ Scalar)

- on R x S^5, ……. (Could Include H=dB)

$$-\frac{i}{2}\bar{\lambda}\Gamma^M\hat{\nabla}_M\lambda-\frac{1}{2}\partial_M\phi_I\partial^M\phi_I-\frac{2}{r^2}\phi_I\phi_I$$

- gamma matrices Γ^M, ρ^a
- Symplectic Majorana λ = -BC λ^*, ε = BC ε^*
- Weyl: $\Gamma^7 \lambda=\lambda$, $\Gamma^7 \varepsilon = -\varepsilon$
- 32 supersymmetry

$$\begin{aligned}\delta\phi_I &= -\bar{\lambda}\rho_I\epsilon=+\bar{\epsilon}\rho_I\lambda,\\ \delta\lambda &= +\frac{i}{6}H_{MNP}\Gamma^{MNP}\epsilon+i\partial_M\phi_I\Gamma^M\rho_I\epsilon-2\phi_I\rho_I\tilde{\epsilon},\\ \delta\bar{\lambda} &= -\frac{i}{6}H_{MNP}\bar{\epsilon}\Gamma^{MNP}+i\partial_M\phi_I\bar{\epsilon}\Gamma^M\rho_I-2\bar{\tilde{\epsilon}}\rho_I\phi_I.\end{aligned}$$

- additional condition on Killing spinor:

$$\hat{\nabla}_M\epsilon=\frac{i}{2r}\Gamma_M\tilde{\epsilon},\quad \Gamma^M\hat{\nabla}_M\tilde{\epsilon}=2i\epsilon,\qquad \tilde{\epsilon}=\pm\Gamma_0\epsilon.$$

Twisting & Dimensional Reduction to R x CP^2

Killing spinor eq $\nabla_M \epsilon_\pm = \pm \frac{1}{2r} \Gamma_M \Gamma_\tau \epsilon_\pm$

- Killing spinors: SO(1,5)=SU(2,2) chiral spinor and 4-dim of Sp(2)=$SO(5)_R$
- 32 Killing spinors = 3x8 (SU(3) triplet) +1x 8 (SU(3) singlet) under SU(3) isometry of CP^2:
  - (I) $\epsilon_+ \sim$ exp(-it/2 +3i y/2)… : singlet
  - (II) $\epsilon_+ \sim$ exp(-it/2 - i y/2)… : triplet
- Twisting

$$\begin{aligned}
\epsilon_{old} &= e^{-\frac{y}{4} M_{IJ} \rho_{IJ}} \epsilon_{new}, \\
\lambda_{old} &= e^{-\frac{y}{4} M_{IJ} \rho_{IJ}} \lambda_{new}, \\
(\phi_1 + i\phi_2)_{old} &= e^{+(3+p)iy/2} (\phi_1 + \phi_2)_{new} \\
(\phi_4 + i\phi_5)_{old} &= e^{+(3-p)iy/2} (\phi_4 + i\phi_5)_{new}.
\end{aligned}$$

$$M_{12} = -M_{21} = \frac{3+p}{2}, \; M_{45} = -M_{54} = \frac{3-p}{2}$$
$$p = \cdots, -5, -3, -1, 1, 3, 5, \cdots .$$

$$\partial_y \to \partial_y + \frac{3i}{2}(R_1 + R_2) + \frac{ip}{2}(R_1 - R_2)$$

$$k \equiv j_1 + j_2 + j_3 + \frac{3}{2}(R_1 + R_2) + \frac{p}{2}(R_1 - R_2), \; p = \text{odd integer}$$

Singlets ϵ_+, ϵ_- for $Q = Q^{++}_{---}, S = Q^{--}_{+++}$

5d Lagrangian

$$Q = Q^{++}_{---}, S = Q^{--}_{+++}$$

- Lagrangian on R x CP^2 with 2 supersymmetries for any p:

$$S = \frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\sqrt{|g|}\,\mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta-\frac{2i}{3}A_\rho A_\sigma A_\eta\Big)$$
$$-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I+\frac{1}{4}[\phi_I,\phi_J]^2-2\phi_I^2-\frac{1}{2}(M_{IJ}\phi_J)^2-i(3-p)[\phi_1,\phi_2]\phi_3-i(3+p)[\phi_4,\phi_5]\phi_3$$
$$-\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda-\frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda]-\frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn}+\frac{1}{8}\bar\lambda M_{IJ}\rho_{IJ}\lambda\Big], \quad (2.27)$$

- Supersymmetry Transformation

$$\delta A_\mu = +i\bar\lambda\gamma_\mu\epsilon = -i\bar\epsilon\gamma_\mu\lambda, \quad \delta\phi_I = -\bar\lambda\rho_I\epsilon = \bar\epsilon\rho_I\lambda,$$
$$\delta\lambda = +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon - 2\phi_I\rho_I\tilde\epsilon - M_{IJ}\phi_I\rho_J\epsilon.$$

- p/2=-1/2 : $k = j_1+j_2+j_3+R_1+2R_2$
  - additional supersymmetries: Total 8 supersymmetries

$$Q^{+-}_{-++},\ Q^{+-}_{+-+},\ Q^{+-}_{++-}\quad \text{conjugates}$$

Coupling Constant Quantization

* Instanton number on CP2

$$\nu = \frac{1}{8\pi^2}\int_{\mathrm{CP}^2} \mathrm{Tr}(F\wedge F) = \frac{1}{16\pi^2}\int_{\mathrm{CP}^2} d^4x\sqrt{|g|}\ \mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

* Instantons represents the momentum K or energy K:

$$\frac{1}{g_{YM}^2} = \frac{K}{4\pi^2 r}$$

* Another approach to quantization: F=2J: 2π flux on a cycle, 1/2 instanton for abelian theory

$$\frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\ \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\partial_\rho A_\sigma A_\eta \Rightarrow K\int dt\ A_0$$

* ‘t Hooft coupling constant: λ= N/K
* Large K => Free Theory

Expected Enhanced Supersymmetries

- Killing spinors with $p/2=-1/2$, $k = j_1+j_2+j_3+ R_1+ 2R_2$
  - k=0: 8 kinds
  - k= ± 1: 14 kinds
  - k= ± 2: 8 kinds
  - k= ± 3: 2 kinds
- # of supersymmetries
  - K ≧ 4: 8 supersymmetries
  - K=3: 10 supersymmetries
  - K=2: 16 supersymmetries
  - K=1: 32 supersymmetries

the index function on S^1 x S^5

- 5d SYM on S^5 Hee-Cheol Kim, Seok Kim:1206.6339; Hee-Cheol Kim, Joonho Kim, S.K. 1211.0144, Minahan-Nedelin-Zabzine, 1207.3763
  - S-dual version of the index
  - Vacuum energy:

$$(\epsilon_0)_{index} = \lim_{\beta' \to 0} \mathrm{Tr}\left[(-1)^F \frac{E-R}{2} e^{-\beta'(E-R_1)}\right]$$
$$= \frac{N(N^2-1)}{6} + \frac{N}{24}$$

- S^1 x CP^2 path integral off-shell
- Stationary phase: $D^1=D^2=0$, $F= 2s J$, $\varphi + D^3=4s$, $s= \mathrm{diag}(s_1,s_2,\ldots,s_N)$
  - analogous to 3-dim Monopole operator
- Path Integral: Off-shell, localization (K=1 case_

$$\sum_{s_1,s_2,\cdots s_N=-\infty}^{\infty} \frac{1}{|W_s|} \oint \left[\frac{d\lambda_i}{2\pi}\right] e^{\frac{\beta}{2}\sum_{i=1}^N s_i^2 - i\sum_i s_i\lambda_i} Z^{(1)}_{\rm pert} Z^{(1)}_{\rm inst} \cdot Z^{(2)}_{\rm pert} Z^{(2)}_{\rm inst} \cdot Z^{(3)}_{\rm pert} Z^{(3)}_{\rm inst} .$$

- For K=1, well-confirmed for k ≤ N with N=1,2,3 with the AdS/CFT calculation

Strange Vacua

- K=1, F=2sJ background

$$U(2)\ (1,-1)$$
$$U(3)\ (2,0,-2),(2,-1,-1),(1,1,-2),(1,0,-1)$$
$$U(4)\ (3,1,-1,-3),(3,1,-2,-2),(2,2,-1,-3),(3,0,-1,-2),$$
$$(2,1,0,-3),(2,0,0,-2),(2,0,-1,-1),(1,1,0,-2),(1,0,0,-1)$$

- the Lowest one $s_G = 2\rho \cdot H$ with negative energy $-2\rho^2$, where ρ = Weyl vector
- Ground State for Index: K ≤ N (Strong 't Hooft coupling λ=N/K)

K	$U(2)$	$U(3)$	$U(4)$	$U(5)$	$U(6)$	$U(7)$	$U(8)$	$U(9)$	$U(N)$
1	-1	-4	-10	-20	-35	-56	-84	-120	$-\frac{N(N^2-1)}{6}$
2	0	-1	-2	-5	-8	-14	-20	-30	
3		0	-1	-2	-3	-6	-9	-12	
4			0	-1	-2	-3	-4	-7	
5				0	-1	-2	-3	-4	
6					0	-1	-2	-3	
7						0	-1	-2	
8							0	-1	
9								0	

Table 1: Vacuum energies divided by K, at general $\mathbb{Z}_K$ (and fluxes)

Check with AdS/CFT

- E.g. k = N = 3: (all results multiplied by vacuum energy factor & $e^{-3\beta}$) $y_i = e^{-\beta a_i}\ ,\quad y = e^{\beta(m-\frac{1}{2})}$

$$\begin{aligned}
Z_{(2,0,-2)} &= 3\left[y^2(y_1+y_2+y_3)+y(y_1^2+y_2^2+y_3^2)+y^{-1}(y_1+y_2+y_3)-(1+\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots)+y^3\right] \\
&\quad +6y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right]+y^3 \\
Z_{(2,-1,-1)}+Z_{(1,1,-2)} &= -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -4y^3-4y^2(y_1+y_2+y_3)-2y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)+2\left(\frac{y_1}{y_2}+\frac{y_2}{y_3}+\cdots\right)-2y^{-1}(y_1+y_2+y_3) \\
Z_{(1,0,-1)} &= y^3+y^2(y_1+y_2+y_3)-y(y_1^{-1}+y_2^{-1}+y_3^{-1})+1
\end{aligned}$$

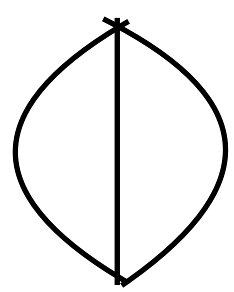
add all

$$Z_{SUGRA} = 3y^3+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^3-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)-\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots\right)+y^{-1}(y_1+y_2+y_3)$$

Non zero flux states contributing to the index

- ✳ s=(N-1,N-3,…,-(N-1)) = s_0 : SU(N) Weyl vector
- ✳ index vacuum energy:

$$E_0 = -\frac{N(N^2-1)}{6}$$



$$f^{abc} f^{abc}$$

Kim &Kim: 2-loop effect on S^5 SYM

SU(2) Case

- BPS Eq. for Homogeneous Configuration with instanton number n^2

$$A = V\mathrm{diag}(n, -n), \;\; F = 2J\mathrm{diag}(n, -n),$$

- homogeneous bosonic solutions possible only with n=+1,-1
- but gauss law is violated
- for one of the constant bps solutions, the homogeneous fermionic zero mode is possible.
- gauss law can be satisfied with fermionic contribution for K=1 but not for K>1.
- energetic is more complicated to due to zero-point contribution to the classical one,...

Conclusion

- New 5d supersymmetric theories for M5 are found with discrete coupling constant and a weak coupling limit
  - Index Function of 6d A_N (2,0) is partially obtained.
  - highly nontrivial vacuum structure in the strong coupled regime
  - UV finite? How rigid is the theory with eight supersymmetries.
  - Enhanced supersymmetry to K=1,2,3?
  - Wilson-loop can be easily included.
  - Near BPS objects? perturbative approach?
- Some effective Lagrangian can be understood in 5d context. No need for magnetic sector.
- 5d formalism ✓ , N^3 ✓

- How to remedy 5d R^{1+4} Super Yang-Mills theory?
- 6-dim (1,0) theories?
- S^6 partition function?