5 and 6 dimensional superconformal field theories

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Recent development in theoretical physics, KIAS
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partly based on works with…

Hee-Cheol Kim, Joonho Kim, Sungsoo Kim, Eunkyung Koh, Kimyeong Lee, Sungjay Lee, in various different combinations (during 2011 - 2013)

Chiung Hwang, Joonho Kim, S.K., Jaemo Park, to appear soon
More work in early progress

Quantum field theories in d > 4

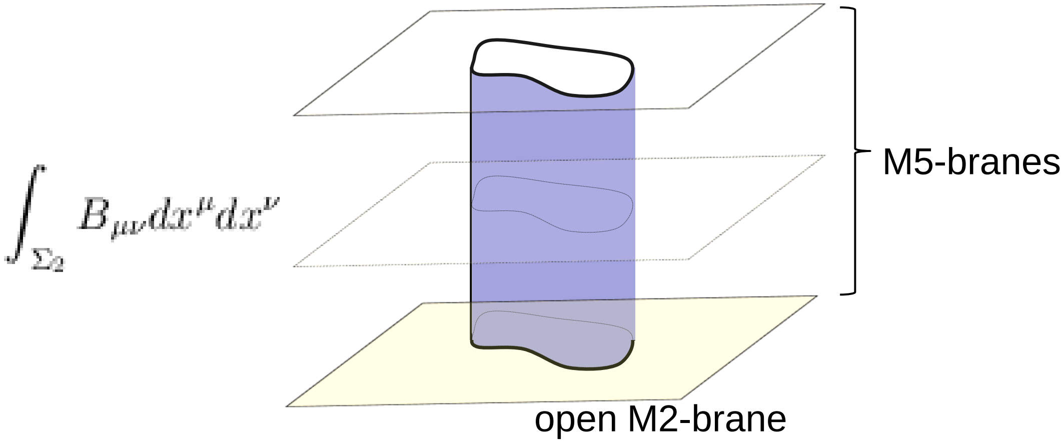
- It is hard from our “textbook QFT” knowledge to imagine interacting QFT in d > 4.
- This is not because we’ve been uninterested They are hard to understand from conventional QFT techniques.

- Local conformal field theories are predicted in d=5,6 from string theory.
- Actually, superconformal field theories. [Witten] (1995 -), [Seiberg et.al.] (1996 -)
- Don’t have known Lagrangian descriptions: beyond “standard” QFT techniques.
- However, expected to have very novel properties, and in some sense “useful.”

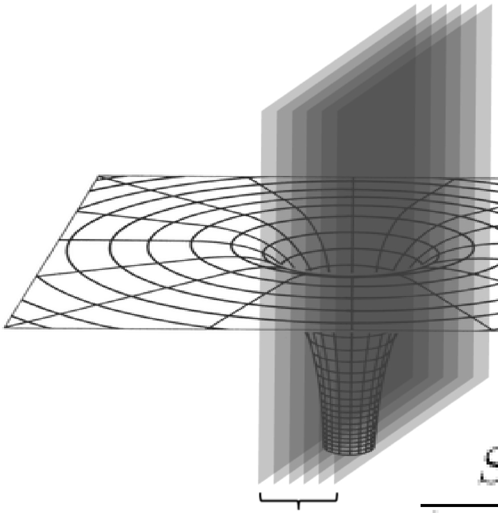
- I will explain novel aspects of these QFT’s, predicted by string theory.
- Also will explain recent studies on their BPS observables using effective field theory descriptions of 5d Yang-Mills.

6d SCFT: “the (2,0) theory”

- It can either be engineered by branes or geometry: e.g. multiple M5-branes.
- “tensor gauge theory” coupling to “self-dual strings”
- N M5-branes host N^3 light degrees:


$$\int_{\Sigma_2} B_{\mu\nu} dx^\mu dx^\nu$$

black M5-brane at temperature T

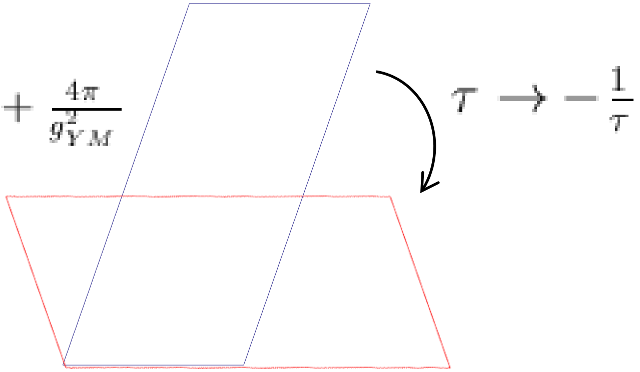


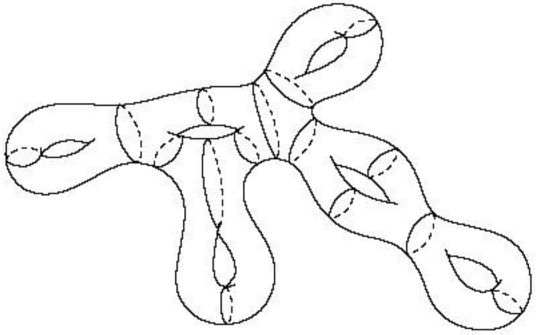
$$\frac{S_{BH}}{(\text{volume})_5} \sim N^3 T^5$$

- Its mere existence leads to interesting predictions on lower dimensional physics.

complex structure $\tau = \frac{\theta}{2\pi} + \frac{4\pi}{g^2_{YM}}$

coupling constant of d=4
maximal super Yang-Mills


$$\tau \rightarrow -\frac{1}{\tau}$$



5d SCFT

- Also engineer by low E decoupling limits in various string backgrounds.
- Uses D-branes , M-theory on CY3, …

- A simplest example is engineer on D4-branes probing N_f D8-O8 system.
- Tuning dilaton: obtains strongly coupled 5d SCFT. (strong coupling limit of 5d SYM on D4)

- Again, many mysterious properties:

\- $N^{5/2}$ light degrees:

\- Possesses infinitely many massless “particles”.

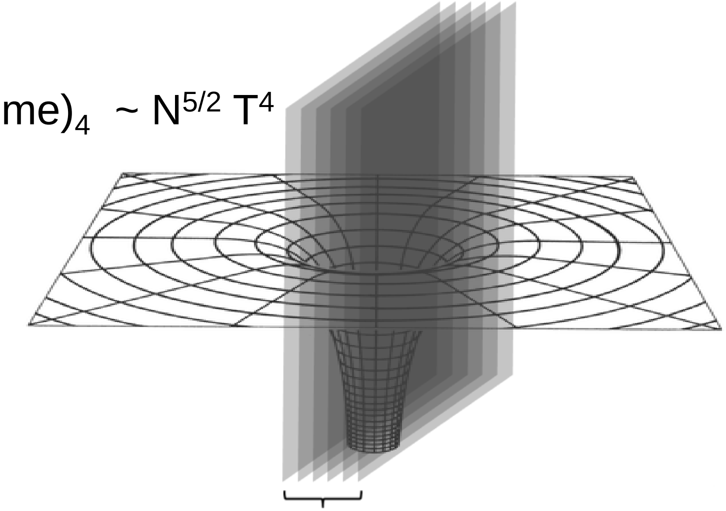
(soiltons of 5d SYM become massless)

\- Possesses tensionless strings

(monopole strings of 5d YM becomes tensionless)

black D4-brane in D8-O8 background, at temperature T: black branes in AdS6

$S_{BH}/(\text{volume})_4 \sim N^{5/2} T^4$



N >>1 D4-branes

- Equally mysterious/interesting as the 6d SCFTs, but somehow less studied…

5 dimensional super-Yang-Mills

- In both 5d/6d, Yang-Mills theory at low E after suitable deformations of CFTs.
- 5d: Yang-Mills kinetic term is a relevant deformation of 5d SCFT. $-\frac{1}{g_{YM}^2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$

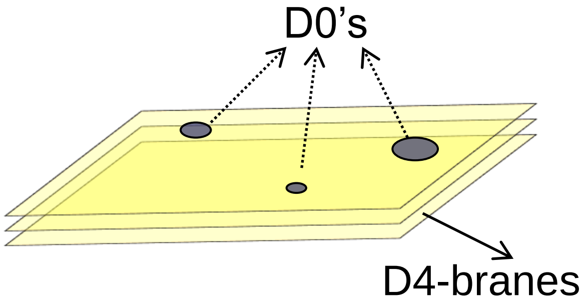
  (inverse coupling is a massive parameter of CFT, like masses for symmetries)
- 6d: circle compactify, "dualize" to vector. 5d maximal SYM (M5's on circle yields D4's)

  (inverse coupling is the radius of the 6th circle: $\frac{4\pi^2}{g_{YM}^2} = \frac{1}{r_1}$)

- Instantons: key non-perturbative object of 5d SYM, provide information on CFT's

$$F_{\mu\nu} = \pm \star_4 F_{\mu\nu} \quad \text{on } R^4 \qquad\qquad k = \frac{1}{8\pi^2}\int \mathrm{tr}(F \wedge F) \in \mathbb{Z}$$

$$E = \frac{1}{4g_{YM}^2}\int d^4x\mathrm{tr}(F_{\mu\nu}^2) = \frac{1}{8g_{YM}^2}\int d^4x\mathrm{tr}\left(F_\mu \mp \star_4 F_{\mu\nu}\right)^2 \pm \frac{1}{2g_{YM}^2}\int \mathrm{tr}(F\wedge F) \geq \frac{4\pi^2|k|}{g_{YM}^2}$$



- 6d: instantons = 6d KK modes.
- 5d: instantons are massless in UV CFT. responsible for UV enhanced symmetry.

- BPS observables for instantons: explore UV CFT from SYM.

Instanton solitons & their indices

- Consider 5d SYM w/ N=1 SUSY in the Coulomb branch. (will be relaxed later)
- Half-BPS marginal bound states of W-bosons & instantons

$$\{Q^A_M, Q^B_N\} = P_\mu(\Gamma^\mu C)_{MN}\epsilon^{AB} + i\frac{4\pi^2 k}{g^2_{YM}}C_{MN}\epsilon^{AB} + i\mathrm{tr}(\Pi v)\epsilon^{AB}C_{MN}$$

instanton number

electric charge

Coulomb VEV

$$M = \frac{4\pi^2 k}{g^2_{YM}} + \mathrm{tr}(qv)$$

preserves $Q^A_{\dot\alpha}$

- These bound states become ¼-BPS in 5d maximal SYM (preserve 4 SUSY)
- Witten index for these BPS particles

$$Z_{\mathrm{Nek}} = \mathrm{Tr}\left[(-1)^F q^k e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_+\left(2J^3_R+J_1+J_2\right)} e^{-\epsilon_-(J_1-J_2)} e^{-\mathrm{tr}(v\Pi)} (\text{flavor fugacities})\right]$$

$Q = -\bar{Q}^2_{\dot 2} = \bar{Q}^{2\dot 1}$, $Q^\dagger = \bar{Q}^1_{\dot 1} = \bar{Q}_{2\dot 1}$

J^3_R: Cartan of $SU(2)_R$, J_1, J_2: $SO(4)$ rotation

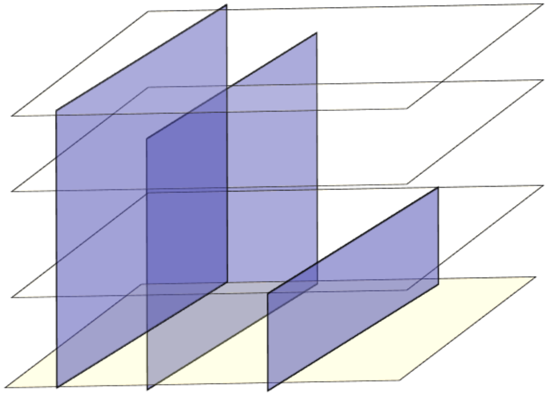
$\epsilon_\pm \equiv \frac{\epsilon_1 \pm \epsilon_2}{2}$

q: electric charge , v: its chemical potential

- This is Nekrasov's partition function for multi-instantons on Ω-deformed R^4 x S^1.

6d (2,0) SCFT

- Partition function of maximal SYM $Z[R^4 \times S^1] = Z[R^4 \times T^2]$: instantons ~ KK modes…?
- Expand it in the fugacity of electric charges: W-bosons in 5d bound to instantons uplifts to the "self-dual strings" for M2 ending on M5



- The 2d index nature of Z_{Nekrasov} in W-boson expansion:

E.g. for SU(2) SYM, with 1 Coulomb VEV, one finds

$$Z_{\text{inst}}^{U(1)} = \sum_Y e^{-\mu_{12}|Y|} \prod_{(i,j)} \frac{\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(i-1)+\epsilon_++m}{2\pi i};q\right)\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(i-1)+\epsilon_++m}{2\pi i};q\right)}{\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(Y_j^T-i)-\epsilon_2}{2\pi i};q\right)\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(Y_j^T-i)+\epsilon_1}{2\pi i};q\right)}$$

[H.-C.Kim, S.K. E. Koh, K. Lee, S. Lee] (2011), [Haghighat, Iqbal, Kozcaz, Lockhart, Vafa] (2013)

- Z_{Nekrasov} ~ self-dual string's elliptic genus index (also: prof. Dongsu Bak's talk)
- It shows that Z_{Nekrasov} correctly captures UV CFT physics. (at least BPS sector)

5d SCFTs

- Computes the BPS spectrum of 5d CFT in the Coulomb phase.
- One has a large class of 5d CFT's known, with 8 SUSY.
- In fact, Z_{nek} for general gauge group & matter contents are not precisely known.

- The Witten index of ADHM instanton quantum mechanics is given by a contour integral: fixing the contour is the tricky part, which we now can. [C. Hwang, J. Kim, S.K., J. Park] to appear soon. See also prof. Piljin Yi's talk
- This index can be used to study various properties of the UV CFT.
- E.g. symmetry enhancements at the UV fixed points, with massless instantons.

- Example: 5d SCFT on D4-D8-O8 system: Sp(N) SYM w/ N_f hypermultiplets
- SO($2N_f$) global symmetry combines w/ instantons' topological charge and enhances to E_{Nf+1} . [H.-C.Kim, S.S.Kim, K. Lee] (2012), [Hwang, Kim, S.K. Park] (to appear) Also studied by [Hayashi, H.-C. Kim, Nishinaka] [Bao, Mitev, Pomoni, Taki Yagi] (2013)

Curved space partition functions from instantons

- $Z_{\rm Nek}$ is also a building block for various curved space partition functions.
- $Z[S^5]$: [H.-C. Kim, SK], [Lockhart, Vafa], [H.-C. Kim, J. Kim, SK] (2012) etc.

$$Z_{S^5}[\beta=\frac{g_{YM}^2}{2\pi r}, m, \phi, r] = \int [d\phi] e^{-\frac{2\pi^2 \mathrm{tr}(\phi^2)}{\beta\omega_1\omega_2\omega_3}} Z_{\rm Nek}^{\mathbb{R}^4\times S^1}\left(q=e^{-\frac{4\pi^2}{\beta\omega_1}}, \epsilon_1=\frac{\omega_2-\omega_1}{\omega_1}, \epsilon_2=\frac{\omega_3-\omega_1}{\omega_1}, \frac{m}{\omega_1}, \frac{\phi}{\omega_1}\right) Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(2) Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(3)$$

- With $Z[R^4 \times S^1]= Z[R^4 \times T^2]$, extra circle makes Z[S5] =Z[S5 x S1]: “superconformal index”
- New predictions on the BPS spectrum of 6d SCFT
- N^3 scalings of the Casimir energy in the index: e.g. 1-parameter index from maximal SYM

$$Z^{U(N)} = e^{\beta\left(\frac{N(N^2-1)}{6}+\frac{N}{24}\right)} \prod_{n=0}^{\infty}\prod_{s=1}^{N}\frac{1}{1-e^{-\beta(n+s)}}$$

- $Z[S^4 \times S^1]$: [H.-C.Kim, S.S.Kim, K.Lee] [Vafa] (2012) [Hwang, SK, J.Kim, Park] (to appear) etc.

$$Z_{S^4\times S^1}[x=e^{-\epsilon_+}, y=e^{-\epsilon_-}, m_i, q] = \int [d\alpha] Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(q, \epsilon_{1,2}, m_i, \alpha) Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(q^{-1}, \epsilon_{1,2}, m_i, \alpha)$$

This is actually the observable with which we studied the E_{N_f+1} symmetry enhancement.

Concluding remarks

- Instanton partition function is a useful observable to understand 5d/6d SCFTs.

- Also conceptually, instantons are objects which show non-particle nature, so very likely to tell us the tricky nature of these higher dimensional QFTs.

- Instanton scale moduli:

- small instanton singularity (UV incomplete of 5d SYM): removable by UV completion
- large instantons (lifted in Coulomb branch): provides continuum from internal degrees.

$$U(2)\ :\ d\lambda^2+\frac{\lambda^2}{4}\left[d\theta^2+\sin^2\theta d\phi^2+(d\psi+\cos\theta d\phi)^2\right]\to\frac{1}{\sqrt{1+\frac{4\zeta^2}{\lambda^4}}}\left[d\lambda^2+\frac{\lambda^2}{4}(d\psi+\cos\theta d\phi)^2\right]+\frac{\lambda^2}{4}\sqrt{1+\frac{4\zeta^2}{\lambda^4}}\left[d\theta^2+\sin^2\theta d\phi^2\right]$$

$\boxed{\mathbb{R}^4/\mathbb{Z}_2}$

$\boxed{\text{Eguchi-Hanson metric: } \mathbb{R}^2\times S^2 \text{ near } \lambda=0}$

- Single instanton cannot be single particle…

- Demands a more universal framework on QFT, not relying on Lagrangian. (large N holography, conformal bootstrap, …)