# M5 brane theory

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- A direct formulation & definition of the M5 brane theory?
- One interesting proposal is [ Douglas, Lambert, Papageogakis, Schmidt-Sommerfeld]

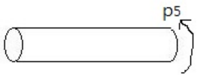
$$\text{M5 brane on M theory circle } S^1 \leftrightarrow \text{5d MSYM of D4}$$

5d SYM coupling

$$g^2 = 4\pi^2 R_5$$

The 5d MSYM theory involves solitonic sector of instanton particles. $\rightarrow$ D0 branes bound to the D4 branes $\leftrightarrow$ KK spectra along the M theory circle direction!

$$p_5 = \frac{k}{R_5}$$



# Troubles

- The 5d MSYM theory is perturbatively non renormalizable since the coupling is dimensionful.
- Even with the maximal number of supersymmetries, the theory involves infinities at six loop orders. [Bern et al] Therefore the theory is NOT quantum finite.
  $\rightarrow$ Problematic!
- One should look for the alternative definitions!
- $\leftarrow$ The proposal includes something nontrivial!

# U(1) M5/D4 correspondence

- First look at the U(1) case which is basically free theory. So one can compute the both sides of the proposals and show that the partition functions agree with each other.
- Look at some details of the agreement:

$$\begin{array}{lcl} \text{M5 on } T^6 = T^5 \times S^1 & & \text{6D } (2,0) \text{ theory} \\ \leftrightarrow \quad \text{5d MSYM on } T^5 & & \text{D4 worldvolume theory} \end{array}$$

- Showing the equivalence is composed of 3 steps.
  1. Zero mode part
  
  The zero mode part is straightforward in principle. But L Dolan and Y Sun found some mismatches. But the trouble was that if one uses the Hamiltonian formulation in the M5 side, one has to use the same Hamiltonian formulation in the D4 side as well.

# U(1) M5/D4 correspondence

Then one finds an agreement.

$$Z_{6d}^{\text{zero}} = Z_{5d}^{\text{zero}}$$

2. Oscillator part

One can see that the number of degrees agree with each other.

$$\begin{array}{llcccccc} \text{M5} & \text{6d} & 3_B\ (\text{SD two form}) & + & 5_S & + & 8_F \\ & & \left(\frac{4\times 3}{2}\cdot\frac{1}{2}=\frac{6}{2}=3\right) & & & & \\ \text{D4} & \text{5d} & 3_A(\text{gauge potential}) & + & 5_S & + & 8_F \end{array}$$

But the 6d/5d oscillator contributions cannot be matching because the KK contribution in the M theory circle direction is completely missing in the 5d side.

$$Z_{6d}^{\text{osc}}(p_5 = 0) = Z_{5d}^{\text{osc}}$$

# U(1) M5/D4 correspondence

3. 5d small instanton contributions

These small instanton configurations are singular. This instanton, which is the D0, corresponds to the KK momentum along the M theory circle direction. One can regularize it by introducing noncommutativity

$$[x_i, x_j] = i\theta_{ij}$$

which makes the size of instanton finite.

By these instanton contributions, the nonvanishing KK contribution to the M5 brane partition function is precisely reproduced:

$$Z_{6d}^{\rm osc}(p_5 \neq 0) = Z_{5d}^{\rm instanton}$$

$\rightarrow$ U(1) M5/D4 correspondence!

# Candidate definitions

- For the nonabelian theory, we do not have a completely satisfactory definition of M5 or D4 brane theory.
- There are some candidate proposals until now:
  1. Deconstruction of dimension based on the $\mathcal{N}=2$ superconformal quiver gauge theories [Arkani-Hamed et al]
  2. $\mathcal{N}=8$ DLCQ quantum mechanics [Aharoney et al]
- Discuss TST duality which is very important consistency of 5d MSYM theory if one accepts the M5/D4 correspondence.

# $\mathcal{N}$=8 DLCQ quantum mechanics Aharony et al

- The $\mathcal{N}$=8 quantum mechanics is well defined for the computation and good even for the description of the non BPS states.
- The DLCQ description of the k D0 brane dynamics is described by that of the $\mathcal{N}$=8 quantum mechanics on the moduli space of the k instantons based on the ADHM construction of the 5d MSYM theory.
- Our $\mathcal{N}$=8 quantum mechanics involves a potential arising from turning on the scalar vev

$$\phi_0 = \langle \phi_9 \rangle$$

This will introduce another mass scale in addition to $R_5$.

# DLCQ limit of 5d MSYM theory

For the k instanton sector, the corresponding KK momentum $p_5$ is given by

$$p_5 = \frac{k}{R_5}$$

and the energy by

$$p^0 = E = \sqrt{p_5^2 + H_\perp}$$

whose leading order contribution is described by the moduli space dynamics of instantons

$$H_\perp = H_{\rm moduli} + O(p_\perp^2 (R_5^2 p_\perp^2)^m (R_5^2 \phi_0^2)^n)$$
$$H_{\rm moduli} = P_\perp^2 + V$$
$$(n + m \geq 1)$$

Let us clarify what happens with the DLCQ limit. As the direction $x^5$ is circle compactified, we have the identification

$$x^5 \sim x^5 + 2\pi R_5$$

and let us boost the system in the $x^5$ direction with a velocity $v$

$$x'^0 = \frac{1}{\sqrt{1-v^2}}(x^0 - v\, x^5)$$
$$x'^5 = \frac{1}{\sqrt{1-v^2}}(x^5 - v\, x^0), \qquad x'^i = x^i$$

Let us further introduce the lightcone coordinates $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^5)$, which transform

$$x'^+ = \epsilon x^+, \quad x'^- = \frac{1}{\epsilon} x^- \quad \leftarrow \quad \epsilon = \sqrt{\frac{1-v}{1+v}}$$

In the limit $v \to 1$, we have the identification

$$({x'}^{+}, {x'}^{-}, {x'}^{i}) \sim ({x'}^{+}, {x'}^{-} + 2\pi R'_5, {x'}^{i}) \leftarrow {R'}_5 = \frac{R_5 \to 0}{\sqrt{2}\epsilon \to 0}$$

and we shall keep $R'_5$ to be finite. This is the DLCQ limit where the moduli space approximation becomes exact. $p'_-$ is carrying the KK momentum with the new radius $R'_5$

$$p'_- = {p'}^{+} = \frac{k}{R'_5} + O(\epsilon^2) \quad \to \quad \frac{k}{R'_5}$$

and our moduli space Hamiltonian becomes exact

$$p'_+ = {p'}^{-} = \frac{R'_5}{2k}(p_\perp^2 + V) + O(\epsilon^2) \quad \to \quad \frac{R'_5}{2k}(p_\perp^2 + V)$$

The anti instanton sector of negative $k$ decouples because their states become infinitely heavy in the DLCQ limit.

# A few more comments

- The resulting DLCQ Hamiltonian is that of the moduli space approximation plus the potential term.
- Its $\mathcal{N}$=8 supersymmetric completion is uniquely fixed by the moduli space metric $g_{AB}$ plus the triholomorphic Killing vector $G$

$$\{g_{AB}, G\} \quad \rightarrow \quad V = G_A G^A$$

- This describes the Coulomb branch dynamics of k D0 branes in the presence of the N parallel D4 branes.
- Due to the potential, we do not have any problem of possible massless modes since the potential is strictly confining.
- There is no interaction between different k sectors as is the usual in the lightcone frame dynamics. Therefore each k sector can be studied separately.

# Our claim

The resulting $\mathcal{N}$=8 quantum mechanics describes the KK sector of the circle compactified M5 branes in the DLCQ limit.

Our results for the k=1 instanton sector:

We test our $\mathcal{N}$=8 quantum mechanics by identifying

$$\text{all of the 1/4 BPS states}$$

and by computing the 1/4 BPS index partition function

$$Z^{\text{index}}_{1/4\ \text{BPS}}$$

We compare this with the result from the localization computation of 5d MSYM theory [Kim et al]

$$Z_{1/4\ \text{MSYM}} \quad (= Z^{\text{index}}_{1/4\ \text{BPS}})$$

which is the Neckrasov partition function and find a perfect agreement!

# TST duality

- One of the most important consistency check of the M5/D4 correspondence is the (T)S(T)-duality of the system. [Tachikawa]
- 1. Start from one configuration.
  2. Perform a chain of dualities which leads to another related configuration.
  3. Ask if one gets the same counting of states from the both configurations.
- This will be a very important consistency check of the correspondence since M5 brane compactified on $T^2$ which leads to the D3, which is invariant under the S duality symmetry basically as the moduli transformation

$$\tau \quad \to \quad -\frac{1}{\tau}$$

of the $T^2$. ← Symmetry of M5 on $T^2$

We begin with the dyonic instanton BPS states

| | | 0 | 1 | 2 | 3 | $\hat{4}$ | $\hat{5}$ | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $D4$ | $x$ | $x$ | $x$ | $x$ | $x$ | | | | | | |
| $k$ | $D0$ | $x$ | | | | | | | | | | |
| | $F1$ | $x$ | | | | | | $x$ | | | | |

The projections of the supersymmetry transformation are given by

$$\Pi_{D4} = \frac{1 + \Gamma_{012345}}{2}, \quad \Pi_{D0} = \frac{1 + \Gamma_{05}}{2}, \quad \Pi_{F1} = \frac{1 + \Gamma_{065}}{2}$$

Since these commute with each other, we shall have 4 remaining supersymmetries after the projection down to the 1/4 BPS states. We shall consider the case where the 4th direction is circle compactified.

# TST duality

First apply a T duality along 4th circle direction.

| | | 0 | 1 | 2 | 3 | $\hat{4}$ | $\hat{5}$ | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $D3$ | $x$ | $x$ | $x$ | $x$ | | | | | | | |
| $k$ | $D1$ | $x$ | | | | | $x$ | | | | | |
| | $F1$ | $x$ | | | | | | $x$ | | | | |

$\rightarrow$ 1/4 BPS dyonic caloron!

Now we apply the S-duality.

| | | 0 | 1 | 2 | 3 | $\hat{4}$ | $\hat{5}$ | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $D3$ | $x$ | $x$ | $x$ | $x$ | | | | | | | |
| $k$ | $F1$ | $x$ | | | | $x$ | | | | | | |
| | $D1$ | $x$ | | | | | | $x$ | | | | |

$\rightarrow$ 1/4 BPS dyons!

# TST duality

Finally we take the T duality again along the 4th circle direction.

| | | 0 | 1 | 2 | 3 | $\hat{4}$ | $\hat{5}$ | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $D4$ | x | x | x | x | x | | | | | | |
| $k$ | $P$ | x | | | | | | | | | | |
| | $D2$ | x | | | | x | | x | | | | |

The related projections are for $D4$, $P$ and $D2$,

$$\Pi_{D4} = \frac{1+\Gamma_{012345}}{2}\,, \quad \Pi_P = \frac{1+\Gamma_{04}}{2}\,, \quad \Pi_{D2} = \frac{1+\Gamma_{046}}{2}$$

These commute with each other. $\rightarrow$ 1/4 BPS states.

# Elliptic genus

- The low energy dynamics of D2 is described by the 2d (4,4) nonlinear sigma model with a target of monopole moduli space. $\leftarrow$ Monopole string problem!
- For the U(3) case of three D4 branes, the target space is given by $R^3(123) \times S^1(\text{gauge circle}) \times \text{TN}$ [Lee, Weinberg, Yi] where

$$ds^2_{\text{TN}} = V d\vec{x}^2 + V^{-1}(d\chi + \vec{A}\cdot d\vec{x})^2$$
$$V = 1 + \frac{R}{r}, \quad \nabla \times \vec{A} = \nabla V$$

- The F1 stretched along the 6th direction corresponds to the electric charge described by the overall $S^1$ momentum together with an U(1) momentum of TN space.

# Elliptic genus for the noncompact spaces

- Dyonic instantion index $\leftrightarrow$ Elliptic genus of the (4,4) NLSM

$$Z(\tau, z) = \mathrm{tr}(-)^F e^{2\pi i z J_0} q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{c}{24}}$$

  [Witten, Eguchi,$\cdots$]
- One can show that the TST duality works!
- Elliptic genus of noncompact space like TN space or EH space [Eguchi, Sugawara, Taromina, $\cdots$]

# Deconstruction of dimensions

- Finally the definition of M5 brane theory based on the deconstruction of the dimension is the best at the moment. It is based on the 4d $\mathcal{N}=2$ superconformal SU$^M$(N) circular quiver gauge theory. [Arkani-Hamed, Lambert et al]
- One can deconstruct 4-th circle direction whose low energy physics perfectly agrees with the 5d MSYM theory.
- Due to S-duality of the $\mathcal{N}=2$ quiver gauge theory, the KK spectra along the 5th direction are automatically reproduced leading to the automatic deconstruction of the 5th direction at the same time. This is quite consistent with the required properties of M5 brane theory.
- One can apply localization. But the subtlety is that the deconstruction requires Higgs branch vev. Therefore at least localization on $T^4$ is possible. ← Actual comparison with D4!

Thank you very much!