4d $\mathcal{N} = 1$ theory from M5 brane

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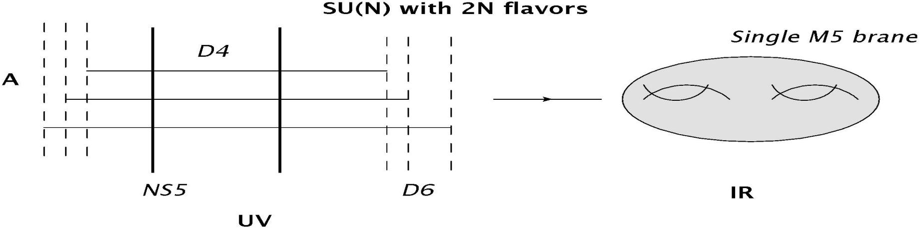
Motivation

- One of main question about dynamics of quantum field theory is: Given a UV theory, what is the IR behavior such as phases, symmetries, effective action, and soliton spectrum, etc.
- Various strong-weak coupling duality such as S-duality of 4d $\mathcal{N} = 4$ and $\mathcal{N} = 2$ theory, Seiberg duality, 3d Mirror symmetry play important role.
- M5 brane (6d (2,0) theory) helps a lot in answering these questions for 4d $\mathcal{N} = 2$ theory: IR solution by Witten, and S duality by Gaiotto, etc.

I am trying to understand Seiberg duality and other dynamical properties of $\mathcal{N} = 1$ theory using M5 branes. In the process, I will find lots of new theories and new mathematical structure.

M5 brane was used very successfully for $\mathcal{N} = 2$ theory in 90s (Witten, 1997). The basic story can be summarized as:
Step 1: Engineer UV theory using type II intersecting brane systems, and identify all the parameters (relevant and marginal deformations). Quantum effects could be seen geometrically too.
Step 2: In the IR limit, the above intersecting brane system becomes a single M5 brane wrapped on a Riemann surface which is identified as the Seiberg-Witten curve.



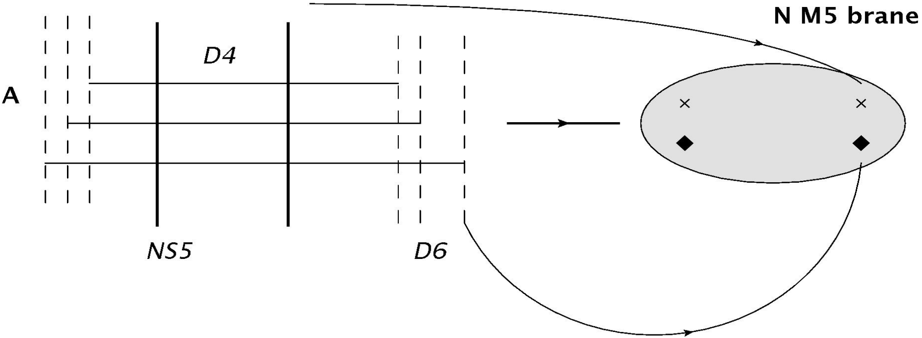
There are some limitations about this approach:

- UV theory usually has to have a Lagrangian description, and strongly coupled theory is hard to deal with.
- String theory understanding of intersecting brane system is poor and many questions about the UV theory are not easy to understand. In particular, the S-duality is not clear as it requires moving branes around which raises all kinds of phase transition questions.

The use of multiple M5 brane: 2009

Instead, one could lift type IIA system for UV theory to a multiple M5 brane system (Gaiotto: 2009):

- D4 branes become N M5 brane wrapped on a compact Riemann surface.
- NS and D6 branes become co-dimensional two defects sitting at Riemann surface.



Surprises

There are many surprises by using this type of engineering:

- By representing the intersections as the defects, you can move them in an arbitrary way without seeing any singularity. So S duality is manifest: they corresponds to different pants decomposition.
- Systematical way of finding Seiberg-Witten curve: spectral curve of Hitchin moduli space.
- Strongly coupled theory can be easily engineered.
- Various physical observables such as BPS particles, extended objects and partition function can be studied exactly using wonderful and well understood geometry and topology of Riemann surface: it links 4d world to 2d world (AGT, etc).
- ....

A new way of engineering 4d theory using M5 brane

This also opens a door for engineering new UV complete theories:

- Start with 6d $(2,0)$ theory and compactify (using topological twist) it on a Riemann surface $M_{g,n}$ with defects, the BPS equation is the so-called Hitchin equation (the Higgs field Φ is living in cotangent bundle)

$$D_{\bar{z}}\Phi = 0, \quad F_{z\bar{z}} + [\Phi, \Phi^*] = 0 \tag{1}$$

- Defects are interpreted as the local singular solution to Hitchin's equation, i.e.

$$\Phi = \frac{e}{z} + \ldots \tag{2}$$

with e nilpotent and labeled by Young Tableaux. The story of irregular defects is extremely rich!

Using these basic building block, one can engineer almost all the interesting four dimensional $\mathcal{N} = 2$ theory: superconformal gauge theory, Argyres-Douglas theory, and asymptotical free theory.

$\mathcal{N} = 1$ and missing ingredients

It is natural to consider $\mathcal{N} = 1$ extension as people has already did (Benini, Wecht, and Tachikawa 2009, I. Bah, C. Beem, N. Bobev, B.Wecht, 2012), however, there are important missing ingredients:

- The M5 brane construction is not explored in full detail, in particular, BPS equation has not been proposed.
- The co-dimensional two defects which provide all the richness have not been studied systematically.

I am going to fill these two gaps.

$\mathcal{N} = 1$ compactification

- One can do $\mathcal{N} = 1$ twist of $(2, 0)$ theory, and there are now two scalars living in line bundles of Riemann surface

$$\Phi_1 \in L_a, \Phi_2 \in L_b \tag{3}$$

and $\Phi_1 \bigotimes \Phi_2 = K$, with K the canonical bundle.

- The BPS equation is a generalized Hitchin equation:

$$\begin{aligned} & D_{\bar{z}}\Phi_1 = 0, \quad D_{\bar{z}}\Phi_2 = 0, \\ & [\Phi_1, \Phi_2] = 0 \\ & F_{z\bar{z}} + [\Phi_1, \Phi_1^*] + [\Phi_2, \Phi_2^*] = 0 \end{aligned} \tag{4}$$

Regular defects

We could immediately have the following regular singular solution

$$
\begin{aligned}
&\Phi_1 = \frac{e_1}{z}, \quad \Phi_2 = \frac{e_2}{z}, \quad A_z = \frac{h_1}{z} + \frac{h_2}{z} \\
&[e_1, e_2] = 0, \quad [h_1, h_2] = 0 \\
&[h_1, e_1] = e_1, \quad [h_2, e_2] = e_2
\end{aligned}
\tag{5}
$$

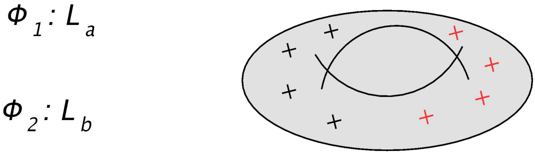
Here e_1, e_2 are nilpotent elements which is called nilpotent pair studied extensively by Ginzburg. Special solutions are $(e_1, 0)$ and $(0, e_2)$, and locally they are just $\mathcal{N} = 2$ punctures. I will focus on them for simplicity, and call them D type puncture and D' type puncture.

Definition of our theory

Our theory is defined by the following data:

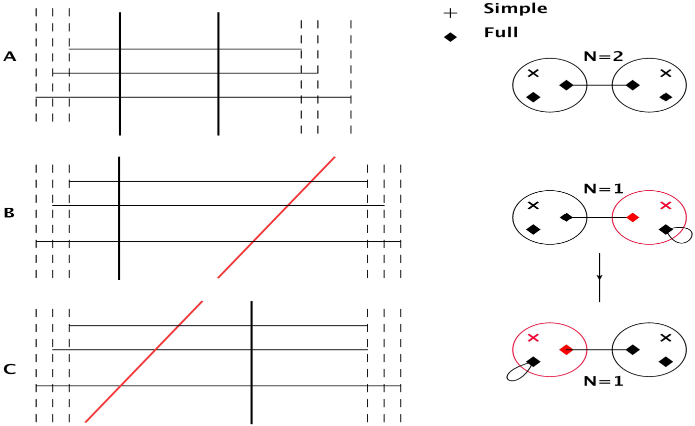
- A Riemann surface $M_{g,n}$.
- A rank two line bundle $L_a \bigoplus L_b$ such that $L_a \bigotimes L_b = K$.
- Various regular D type or $D^{'}$ type punctures.

We conjecture that they are flowing to IR fixed points with $\mathcal{N} = 1$ supersymmetry.



Example: Rotated puncture

It's well known that type IIA brane construction for $\mathcal{N}=1$ theory is derived by rotating $\mathcal{N}=2$ brane system (Kutasov, et al. 1997).



So Seiberg duality is understood also as the degeneration limit of the same Riemann surface much as $\mathcal{N}=2$ duality.

More details

There is one obvious question: which physical parameter is identified with complex structure moduli? In $\mathcal{N}=2$, that is the coupling of cubic superpotential term

$$W = \tau tr(\Phi(\mu_1 - \mu_2)). \tag{6}$$

S-duality exchanges $\tau \to -\frac{1}{\tau}$ which is exactly the same as the change of complex structure moduli. In the current case, we propose that there is a quartic superpotential term

$$W = ctr(\mu_1 \mu_2). \tag{7}$$

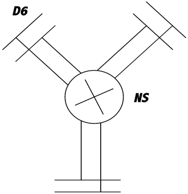
and it is exactly marginal! This coupling has to be there for Seiberg duality to work. Moreover, using the field theory, after duality the superpotential becomes

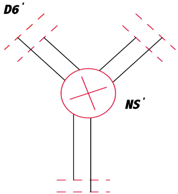
$$W^{'} = \frac{1}{c} tr(\mu_1^{'} \mu_2^{'}) \tag{8}$$

which is in agreement with the geometric interpretation.

General pants decomposition

In fact, there are another $\mathcal{N}=2$ three sphere derived by rotating both D6 brane and NS5 brane.



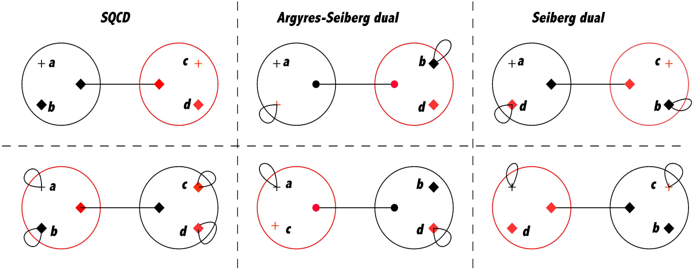


$\mathcal{N}=2$ theory is broken to $\mathcal{N}=1$ in following two ways

- If NS three sphere is connected to a NS' three sphere, one get a $\mathcal{N}=1$ gauge group and quartic superpotential.
- If a D type puncture is connected to a NS' 3 sphere, we get meson and cubic superpotential.

Other duality frames

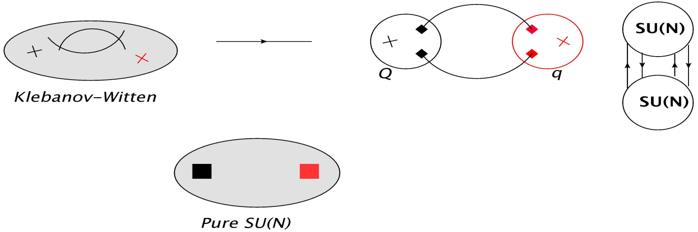
The six duality frames of SQCD:



This explains the duality found in (Tachikawa, et. al.)

More examples

Here is another example: Klebanov-Witten like theory with potential $W = c_1 Q\tilde{Q}q\tilde{q} + c_2\tilde{Q}Q\tilde{q}q$.



And pure SU(N) theory needs rotated irregular singularity.

Conclusion

M5 branes are used to construct a large class of new $\mathcal{N} = 1$ theories. It would be interesting to further study them:

- Study new regular puncture in more detail; irregular singularity story should be very rich, i.e. they are needed for confining theories.
- Study low energy effective theory, index, extended objects, central charges, etc, it seems they can be exactly calculated.
- Compactify 4d theory to 3d and we have $\mathcal{N} = 2$ theory, one can study 3d Seiberg duality and mirror symmetry.
- Any 2d-4d correspondence?
- The moduli space of generalized Hitchin equation is very important and interesting.
- ....