

Top Quark Polarization Observables at Hadron Colliders

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Motivation

General Analysis

Model independent vs. Models

Summary and Prospect

Recent Results @Tevatron ¹

- **CDF, l+jets:**

$$A_{\text{FB}}^{t\bar{t}} = 0.162 \pm 0.042 [0.058 \pm 0.009]$$

- **CDF, dileptons:**

$$A_{\text{FB}}^{t\bar{t}} = 0.42 \pm 0.16$$

- **Combined:**

$$A_{\text{FB}}^{t\bar{t}} = 0.201 \pm 0.067 \quad \text{cf.} \quad A_{\text{FB}}^{t\bar{t}} = 0.196 \pm 0.065 \text{ @ D0}$$

- **CDF, l+jets:**

$$A_{\text{FB}}^{t\bar{t}}(|\Delta y| < 1.0) = 0.088 \pm 0.042 \pm 0.022 [0.043]$$

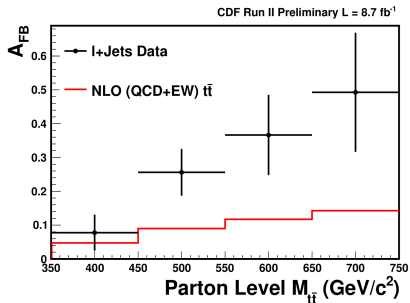
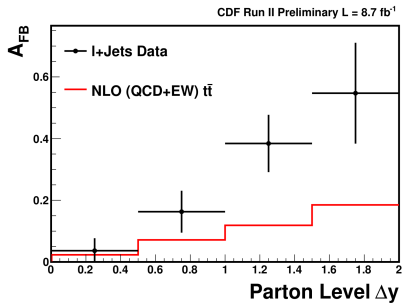
$$A_{\text{FB}}^{t\bar{t}}(|\Delta y| \geq 1.0) = 0.433 \pm 0.097 \pm 0.050 [0.139]$$

- **CDF, l+jets:**

$$A_{\text{FB}}^{t\bar{t}}(M_{t\bar{t}} < 450 \text{ GeV}) = -0.078 \pm 0.048 \pm 0.024 [0.047]$$

$$A_{\text{FB}}^{t\bar{t}}(M_{t\bar{t}} \geq 450 \text{ GeV}) = 0.296 \pm 0.059 \pm 0.031 [0.100]$$

¹The values in the squared bracket are SM predictions. 



etc.,etc,.....

Systematic study of New Physics (NP)

With $q = u, d, s, c, b,$

- Spin-1 NP :

$$\begin{aligned}
 \mathcal{L}_{\text{int}}^{\text{spin-1}} &= g_s \sum_{A=L,R} V_{1A}^\mu \left[g_{1q}^A (\bar{q}_A \gamma_\mu q_A) + g_{1t}^A (\bar{t}_A \gamma_\mu t_A) \right] \\
 &+ g_s \sum_{A=L,R} V_{8A}^{a\mu} \left[g_{8q}^A (\bar{q}_A \gamma_\mu T^a q_A) + g_{8t}^A (\bar{t}_A \gamma_\mu T^a t_A) \right] \\
 &+ g_s \sum_{A=L,R} \left[\tilde{V}_{1A}^\mu \tilde{g}_{1q}^A (\bar{t}_A \gamma_\mu q_A) + \tilde{V}_{8A}^{a\mu} \tilde{g}_{8q}^A (\bar{t}_A \gamma_\mu T^a q_A) + \text{h.c.} \right]
 \end{aligned}$$

- **Spin-0 NP :**

$$\begin{aligned}
 \mathcal{L}_{\text{int}}^{\text{spin-0}} &= g_s \sum_{A=L,R} S_{1A} \left[\eta_{1q}^A (\bar{q} P_A q) + \eta_{1t}^A (\bar{t} P_A t) \right] \\
 &+ g_s \sum_{A=L,R} S_{8A}^a \left[\eta_{8q}^A (\bar{q} T^a P_A q) + \eta_{8t}^A (\bar{t} T^a P_A t) \right] \\
 &+ g_s \sum_{A=L,R} \left[\tilde{S}_{1A} \tilde{\eta}_{1q}^A (\bar{t} P_A q) + \tilde{S}_{8A}^a \tilde{\eta}_{8q}^A (\bar{t} T^a P_A q) + \text{h.c.} \right] \\
 &+ g_s \epsilon_{\alpha\beta\gamma} \left[\tilde{\eta}_{3q} \left(q_R^\alpha t_R^\beta \tilde{S}_3^\gamma \right) + \text{h.c.} \right] \\
 &+ g_s \left[\tilde{\eta}_{6q} \left(q_R^\alpha t_R^\beta \tilde{S}_6^{\alpha\beta} \right) + \text{h.c.} \right]
 \end{aligned}$$

- **Most generally, the amplitude for the process $q\bar{q} \rightarrow t\bar{t}$ can be cast into the form**

$$\begin{aligned}
 \mathcal{M}(q\bar{q} \rightarrow t\bar{t}) &= \frac{g_s^2}{\hat{s}} \sum_{A,B=L,R} \left\{ C_{S1}^{AB} [\overline{v_q(k_2)} P_A u_q(k_1)] [\overline{u_t(p_1)} P_B v_t(p_2)] \right. \\
 &\quad + C_{S8}^{AB} [\overline{v_q(k_2)} P_A T^a u_q(k_1)] [\overline{u_t(p_1)} P_B T^a v_t(p_2)] \\
 &\quad + C_{V1}^{AB} [\overline{v_q(k_2)} \gamma_\mu P_A u_q(k_1)] [\overline{u_t(p_1)} \gamma^\mu P_B v_t(p_2)] \\
 &\quad + C_{V8}^{AB} [\overline{v_q(k_2)} \gamma_\mu P_A T^a u_q(k_1)] [\overline{u_t(p_1)} \gamma^\mu P_B T^a v_t(p_2)] \\
 &\quad + C_{T1}^A [\overline{v_q(k_2)} \sigma_{\mu\nu} P_A u_q(k_1)] [\overline{u_t(p_1)} \sigma^{\mu\nu} P_A v_t(p_2)] \\
 &\quad \left. + C_{T8}^A [\overline{v_q(k_2)} \sigma_{\mu\nu} P_A T^a u_q(k_1)] [\overline{u_t(p_1)} \sigma^{\mu\nu} P_A T^a v_t(p_2)] \right\}
 \end{aligned}$$

where $P_A = (1 + \alpha\gamma_5)/2$ with $\alpha = - (+)$ for $A = L (R)$.

Table: $\tilde{\chi} \equiv \hat{s}/(\hat{x} - M^2 + iM\Gamma)$ with $x = s, t, u$.

Coef.	SM	V_1^μ	$V_8^{a\mu}$	\tilde{V}_1^μ	$\tilde{V}_8^{a\mu}$
$C_{S_1}^{LL}$ $C_{S_1}^{RR}$ $C_{S_1}^{LR}$ $C_{S_1}^{RL}$				$-\frac{2}{N}\tilde{g}_{1q}^L(\tilde{g}_{1q}^R)^* \tilde{t}$ $-\frac{2}{N}\tilde{g}_{1q}^R(\tilde{g}_{1q}^L)^* \tilde{t}$	$-(1 - \frac{1}{N^2})\tilde{g}_{8q}^L(\tilde{g}_{8q}^R)^* \tilde{t}$ $-(1 - \frac{1}{N^2})\tilde{g}_{8q}^R(\tilde{g}_{8q}^L)^* \tilde{t}$
$C_{S_8}^{LL}$ $C_{S_8}^{RR}$ $C_{S_8}^{LR}$ $C_{S_8}^{RL}$				$-4\tilde{g}_{1q}^L(\tilde{g}_{1q}^R)^* \tilde{t}$ $-4\tilde{g}_{1q}^R(\tilde{g}_{1q}^L)^* \tilde{t}$	$\frac{2}{N}\tilde{g}_{8q}^L(\tilde{g}_{8q}^R)^* \tilde{t}$ $\frac{2}{N}\tilde{g}_{8q}^R(\tilde{g}_{8q}^L)^* \tilde{t}$
$C_{V_1}^{LL}$ $C_{V_1}^{RR}$ $C_{V_1}^{LR}$ $C_{V_1}^{RL}$		$g_{1q}^L g_{1t}^L \tilde{s}$ $g_{1q}^R g_{1t}^R \tilde{s}$ $g_{1q}^L g_{1t}^R \tilde{s}$ $g_{1q}^R g_{1t}^L \tilde{s}$		$\frac{1}{N} \tilde{g}_{1q}^L ^2 \tilde{t}$ $\frac{1}{N} \tilde{g}_{1q}^R ^2 \tilde{t}$	$\frac{1}{2}(1 - \frac{1}{N^2}) \tilde{g}_{8q}^L ^2 \tilde{t}$ $\frac{1}{2}(1 - \frac{1}{N^2}) \tilde{g}_{8q}^R ^2 \tilde{t}$
$C_{V_8}^{LL}$ $C_{V_8}^{RR}$ $C_{V_8}^{LR}$ $C_{V_8}^{RL}$	1 1 1 1		$g_{8q}^L g_{8t}^L \tilde{s}$ $g_{8q}^R g_{8t}^R \tilde{s}$ $g_{8q}^L g_{8t}^R \tilde{s}$ $g_{8q}^R g_{8t}^L \tilde{s}$	$2 \tilde{g}_{1q}^L ^2 \tilde{t}$ $2 \tilde{g}_{1q}^R ^2 \tilde{t}$	$-\frac{1}{N} \tilde{g}_{8q}^L ^2 \tilde{t}$ $-\frac{1}{N} \tilde{g}_{8q}^R ^2 \tilde{t}$
$C_{T_1}^L$ $C_{T_1}^R$					
$C_{T_8}^L$ $C_{T_8}^R$					

Coef.	S_1	S_8^a	\tilde{S}_1	\tilde{S}_8^a	\tilde{S}_3^γ	$\tilde{S}_6^{\alpha\beta}$
$C_{S_1}^{LL}$ $C_{S_1}^{RR}$ $C_{S_1}^{LR}$ $C_{S_1}^{RL}$	$-\eta_{1q}^L \eta_{1t}^L \tilde{\xi}$ $-\eta_{1q}^R \eta_{1t}^R \tilde{\xi}$ $-\eta_{1q}^L \eta_{1t}^R \tilde{\xi}$ $-\eta_{1q}^R \eta_{1t}^L \tilde{\xi}$		$\frac{1}{2N} \tilde{\eta}_{1q}^L (\tilde{\eta}_{1q}^R)^* \tilde{t}$ $\frac{1}{2N} \tilde{\eta}_{1q}^R (\tilde{\eta}_{1q}^L)^* \tilde{t}$	$\frac{1}{4} (1 - \frac{1}{N^2}) \tilde{\eta}_{8q}^L (\tilde{\eta}_{8q}^R)^* \tilde{t}$ $\frac{1}{4} (1 - \frac{1}{N^2}) \tilde{\eta}_{8q}^R (\tilde{\eta}_{8q}^L)^* \tilde{t}$		
$C_{S_8}^{LL}$ $C_{S_8}^{RR}$ $C_{S_8}^{LR}$ $C_{S_8}^{RL}$		$-\eta_{8q}^L \eta_{8t}^L \tilde{\xi}$ $-\eta_{8q}^R \eta_{8t}^R \tilde{\xi}$ $-\eta_{8q}^L \eta_{8t}^R \tilde{\xi}$ $-\eta_{8q}^R \eta_{8t}^L \tilde{\xi}$	$\tilde{\eta}_{1q}^L (\tilde{\eta}_{1q}^R)^* \tilde{t}$ $\tilde{\eta}_{1q}^R (\tilde{\eta}_{1q}^L)^* \tilde{t}$	$-\frac{1}{2N} \tilde{\eta}_{8q}^L (\tilde{\eta}_{8q}^R)^* \tilde{t}$ $-\frac{1}{2N} \tilde{\eta}_{8q}^R (\tilde{\eta}_{8q}^L)^* \tilde{t}$		
$C_{V_1}^{LL}$ $C_{V_1}^{RR}$ $C_{V_1}^{LR}$ $C_{V_1}^{RL}$			$\frac{1}{2N} \tilde{\eta}_{1q}^L ^2 \tilde{t}$ $\frac{1}{2N} \tilde{\eta}_{1q}^R ^2 \tilde{t}$	$\frac{1}{4} (1 - \frac{1}{N^2}) \tilde{\eta}_{8q}^L ^2 \tilde{t}$ $\frac{1}{4} (1 - \frac{1}{N^2}) \tilde{\eta}_{8q}^R ^2 \tilde{t}$	$-\frac{1}{2} (1 - \frac{1}{N}) \tilde{\eta}_{3q} ^2 \tilde{u}$	$-\frac{1}{4} (1 + \frac{1}{N}) \tilde{\eta}_{6q} ^2 \tilde{u}$
$C_{V_8}^{LL}$ $C_{V_8}^{RR}$ $C_{V_8}^{LR}$ $C_{V_8}^{RL}$			$ \tilde{\eta}_{1q}^L ^2 \tilde{t}$ $ \tilde{\eta}_{1q}^R ^2 \tilde{t}$	$-\frac{1}{2N} \tilde{\eta}_{8q}^L ^2 \tilde{t}$ $-\frac{1}{2N} \tilde{\eta}_{8q}^R ^2 \tilde{t}$	$ \tilde{\eta}_{3q} ^2 \tilde{u}$	$-\frac{1}{2} \tilde{\eta}_{6q} ^2 \tilde{u}$
$C_{T_1}^L$ $C_{T_1}^R$			$\frac{1}{8N} \tilde{\eta}_{1q}^L (\tilde{\eta}_{1q}^R)^* \tilde{t}$ $\frac{1}{8N} \tilde{\eta}_{1q}^R (\tilde{\eta}_{1q}^L)^* \tilde{t}$	$\frac{1}{16} (1 - \frac{1}{N^2}) \tilde{\eta}_{8q}^L (\tilde{\eta}_{8q}^R)^* \tilde{t}$ $\frac{1}{16} (1 - \frac{1}{N^2}) \tilde{\eta}_{8q}^R (\tilde{\eta}_{8q}^L)^* \tilde{t}$		
$C_{T_8}^L$ $C_{T_8}^R$			$\frac{1}{4} \tilde{\eta}_{1q}^L (\tilde{\eta}_{1q}^R)^* \tilde{t}$ $\frac{1}{4} \tilde{\eta}_{1q}^R (\tilde{\eta}_{1q}^L)^* \tilde{t}$	$-\frac{1}{8N} \tilde{\eta}_{8q}^L (\tilde{\eta}_{8q}^R)^* \tilde{t}$ $-\frac{1}{8N} \tilde{\eta}_{8q}^R (\tilde{\eta}_{8q}^L)^* \tilde{t}$		

- **The amplitude can be decomposed into the colour-singlet and colour-octet parts as**

$$\mathcal{M}(q\bar{q} \rightarrow t\bar{t}) = \frac{g_s^2}{\hat{s}} [\delta_{ij}\delta_{kl} \mathcal{S}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda}) + T_{ij}^a T_{kl}^a \mathcal{O}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda})]$$

- **In the $t\bar{t}$ rest frame,**

$$\mathcal{S}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda}) = \sum_{A,B=L,R} [C_{S1}^{AB} \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB}^S + C_{V1}^{AB} \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB}^V + C_{T1}^A \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_A^T]$$

$$\mathcal{O}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda}) = \sum_{A,B=L,R} [C_{S8}^{AB} \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB}^S + C_{V8}^{AB} \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB}^V + C_{T8}^A \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_A^T]$$

- The reduced amplitudes

$$\langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB}^S = -\frac{\hat{s}}{4} (A + \lambda)(B - \hat{\beta}\sigma) \delta_{\lambda, \bar{\lambda}} \delta_{\sigma, \bar{\sigma}},$$

$$\begin{aligned} \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB}^V &= -\frac{m_t \sqrt{\hat{s}}}{2} (1 + A\lambda) \sigma s_{\hat{\theta}} \delta_{\lambda, -\bar{\lambda}} \delta_{\sigma, \bar{\sigma}} \\ &\quad -\frac{\hat{s}}{4} \left[(1 + A\lambda)(1 + \hat{\beta}B\sigma) c_{\hat{\theta}} + (A + \lambda)(\hat{\beta}B + \sigma) \right] \delta_{\lambda, -\bar{\lambda}} \delta_{\sigma, -\bar{\sigma}}, \end{aligned}$$

$$\langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_A^T = \hat{s} (A + \lambda)(\hat{\beta}A - \sigma) c_{\hat{\theta}} \delta_{\lambda, \bar{\lambda}} \delta_{\sigma, \bar{\sigma}} - 2i m_t \sqrt{\hat{s}} (A + \lambda) s_{\hat{\theta}} \delta_{\lambda, \bar{\lambda}} \delta_{\sigma, -\bar{\sigma}},$$

with $\hat{\beta} = \sqrt{1 - 4m_t^2/\hat{s}}$.

- The squared amplitude

$$\overline{|\mathcal{M}(q\bar{q} \rightarrow t\bar{t})|^2} \equiv \frac{1}{9} \cdot \frac{1}{4} \cdot \frac{g_s^4}{\hat{s}^2} \sum_{\lambda, \bar{\lambda}, \sigma, \bar{\sigma}} [9 |\mathcal{S}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda})|^2 + 2 |\mathcal{O}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda})|^2]$$

using $\sum_{\text{colours}} |\delta_{ij} \delta_{kl}|^2 = 9$, $\sum_{\text{colours}} \delta_{ij} T_{ij}^a = 0$, $\sum_{\text{colours}} |T_{ij}^a T_{kl}^a|^2 = 2$.

- **The matrix element takes the form**

$$\mathcal{A}(\sigma\bar{\sigma}; \lambda\bar{\lambda}) \propto \begin{pmatrix} \langle ++; \lambda\bar{\lambda} \rangle & \langle +-; \lambda\bar{\lambda} \rangle \\ \langle -+; \lambda\bar{\lambda} \rangle & \langle --; \lambda\bar{\lambda} \rangle \end{pmatrix}$$

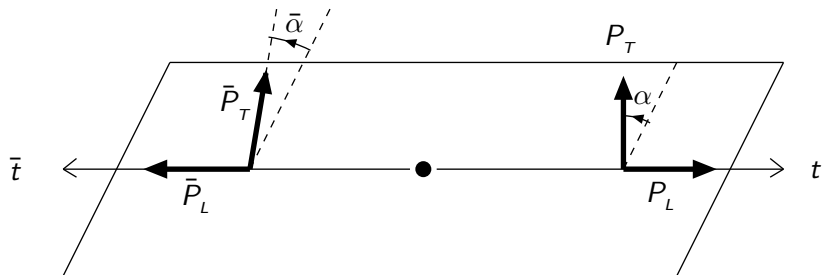
with $\mathcal{A} = \mathcal{S}, \mathcal{O}$.

- **The top polarization-weighted squared matrix elements are given by**

$$\overline{|\mathcal{M}(q\bar{q} \rightarrow t\bar{t})|^2} = \frac{g_s^4}{\hat{s}^2} \sum_{\lambda, \bar{\lambda}} \left\{ \sum_{\mathcal{A}=\mathcal{S}, \mathcal{O}} f_{\mathcal{A}} \text{Tr} [\mathcal{A}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda}) \bar{\rho}^T \mathcal{A}^\dagger(\sigma, \bar{\sigma}; \lambda, \bar{\lambda}) \rho] \right\}$$

with $f_{\mathcal{S}} = 1$ and $f_{\mathcal{O}} = 2/9$ and

$$\rho = \frac{1}{2} \begin{pmatrix} 1 + P_L & P_T e^{-i\alpha} \\ P_T e^{i\alpha} & 1 - P_L \end{pmatrix}, \quad \bar{\rho} = \frac{1}{2} \begin{pmatrix} 1 + \bar{P}_L & -\bar{P}_T e^{i\bar{\alpha}} \\ -\bar{P}_T e^{-i\bar{\alpha}} & 1 - \bar{P}_L \end{pmatrix}$$



The $t\bar{t}$ production plane in its rest frame.

The longitudinal-polarization vector $P_L(\bar{P}_L)$ and the transverse-polarization vector $P_T(\bar{P}_T)$ with the azimuthal angle $\alpha(\bar{\alpha})$ of $t(\bar{t})$ are shown.

- The amplitude squared can be spanned as

$$\begin{aligned}
 |\mathcal{M}|^2 &= \frac{g_s^4}{\hat{s}^2} \left\{ D_0 + D_1(P_L + \bar{P}_L) + D_2(P_L - \bar{P}_L) + D_3 P_L \bar{P}_L \right. \\
 &+ [D_4 \sin(\alpha - \bar{\alpha}) + D_5 \cos(\alpha - \bar{\alpha}) + D_6 \sin(\alpha + \bar{\alpha}) + D_7 \cos(\alpha + \bar{\alpha})] P_T \bar{P}_T \\
 &+ D_8(P_T \sin \alpha + \bar{P}_T \sin \bar{\alpha}) + D_9(P_T \sin \alpha - \bar{P}_T \sin \bar{\alpha}) \\
 &+ D_{10}(P_T \cos \alpha + \bar{P}_T \cos \bar{\alpha}) + D_{11}(P_T \cos \alpha - \bar{P}_T \cos \bar{\alpha}) \\
 &+ D_{12}(P_L \bar{P}_T \sin \bar{\alpha} + \bar{P}_L P_T \sin \alpha) + D_{13}(P_L \bar{P}_T \sin \bar{\alpha} - \bar{P}_L P_T \sin \alpha) \\
 &\left. + D_{14}(P_L \bar{P}_T \cos \bar{\alpha} + \bar{P}_L P_T \cos \alpha) + D_{15}(P_L \bar{P}_T \cos \bar{\alpha} - \bar{P}_L P_T \cos \alpha) \right\}
 \end{aligned}$$

- The coefficients D_i 's are expressed by helicity amplitudes

$$D_0 = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(++; \lambda \bar{\lambda})|^2 + |\mathcal{A}(--; \lambda \bar{\lambda})|^2 + |\mathcal{A}(+-; \lambda \bar{\lambda})|^2 + |\mathcal{A}(-+; \lambda \bar{\lambda})|^2 \right),$$

$$D_1 = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(++; \lambda \bar{\lambda})|^2 - |\mathcal{A}(--; \lambda \bar{\lambda})|^2 \right),$$

$$D_2 = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(+-; \lambda \bar{\lambda})|^2 - |\mathcal{A}(-+; \lambda \bar{\lambda})|^2 \right),$$

$$D_3 = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(++; \lambda \bar{\lambda})|^2 + |\mathcal{A}(--; \lambda \bar{\lambda})|^2 - |\mathcal{A}(+-; \lambda \bar{\lambda})|^2 - |\mathcal{A}(-+; \lambda \bar{\lambda})|^2 \right),$$

$$D_4 = \frac{1}{2} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \Im \mathfrak{m}[\mathcal{A}(++; \lambda \bar{\lambda}) \mathcal{A}^*(---; \lambda \bar{\lambda})],$$

$$D_5 = -\frac{1}{2} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \Re \mathfrak{e}[\mathcal{A}(++; \lambda \bar{\lambda}) \mathcal{A}^*(---; \lambda \bar{\lambda})],$$

$$D_6 = \frac{1}{2} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \Im \mathfrak{m}[\mathcal{A}(+-; \lambda \bar{\lambda}) \mathcal{A}^*(-+; \lambda \bar{\lambda})^*],$$

$$D_7 = -\frac{1}{2} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \Re \mathfrak{e}[\mathcal{A}(+-; \lambda \bar{\lambda}) \mathcal{A}^*(-+; \lambda \bar{\lambda})],$$

$$D_8 = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Im \mathfrak{m}[\mathcal{A}(---; \lambda \bar{\lambda}) \mathcal{A}^*(+-; \lambda \bar{\lambda})] - \Im \mathfrak{m}[\mathcal{A}(++; \lambda \bar{\lambda}) \mathcal{A}^*(-+; \lambda \bar{\lambda})] \right. \\ \left. - \Im \mathfrak{m}[\mathcal{A}(++; \lambda \bar{\lambda}) \mathcal{A}^*(+-; \lambda \bar{\lambda})] + \Im \mathfrak{m}[\mathcal{A}(---; \lambda \bar{\lambda}) \mathcal{A}^*(-+; \lambda \bar{\lambda})] \right),$$

$$D_9 = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Im \mathfrak{m}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] - \Im \mathfrak{m}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right. \\ \left. + \Im \mathfrak{m}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] - \Im \mathfrak{m}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right),$$

$$D_{10} = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Re \mathfrak{e}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] + \Re \mathfrak{e}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right. \\ \left. - \Re \mathfrak{e}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] - \Re \mathfrak{e}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right),$$

$$D_{11} = \frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Re \mathfrak{e}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] + \Re \mathfrak{e}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right. \\ \left. + \Re \mathfrak{e}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] + \Re \mathfrak{e}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right),$$

$$D_{12} = -\frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Im \mathfrak{m}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] + \Im \mathfrak{m}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right. \\ \left. + \Im \mathfrak{m}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] + \Im \mathfrak{m}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right),$$

$$D_{13} = -\frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Im \mathfrak{m}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] + \Im \mathfrak{m}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right. \\ \left. - \Im \mathfrak{m}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] - \Im \mathfrak{m}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right),$$

$$D_{14} = -\frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Re \mathfrak{e}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] - \Re \mathfrak{e}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right. \\ \left. + \Re \mathfrak{e}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] - \Re \mathfrak{e}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right),$$

$$D_{15} = -\frac{1}{4} \sum_{\lambda, \bar{\lambda}, \mathcal{A}} f_{\mathcal{A}} \left(\Re \mathfrak{e}[\mathcal{A}(+; \lambda \bar{\lambda}) \mathcal{A}^*(+; \lambda \bar{\lambda})] - \Re \mathfrak{e}[\mathcal{A}(-; \lambda \bar{\lambda}) \mathcal{A}^*(-; \lambda \bar{\lambda})] \right)$$

Table: The CP and CPT \tilde{T} parities of the polarization coefficients.

Observables	[CP, CPT \tilde{T}]
D_0	[+, +]
D_1	[-, -]
D_2	[+, +]
D_3	[+, +]
D_4	[-, +]
D_5	[+, +]
D_6	[+, -]
D_7	[+, +]
D_8	[+, -]
D_9	[-, +]
D_{10}	[+, +]
D_{11}	[-, -]
D_{12}	[-, +]
D_{13}	[+, -]
D_{14}	[-, -]
D_{15}	[+, +]

Example : EFT analysis

- If the NP particles are heavy enough, we can adopt the viewpoint of effective Lagrangian.
- Assuming $SU(2)_L \times U(1)_Y$ and the custodial symmetry $SU(2)_R$ for the light quark sector,

$$\mathcal{L}_6 = \frac{g_s^2}{\Lambda^2} \sum_{A,B} \left[C_{1q}^{AB} (\bar{q}_A \gamma_\mu q_A) (\bar{t}_B \gamma^\mu t_B) + C_{8q}^{AB} (\bar{q}_A T^a \gamma_\mu q_A) (\bar{t}_B T^a \gamma^\mu t_B) \right]$$

where $T^a = \lambda^a/2$, $\{A, B\} = \{L, R\}$, and $L, R \equiv (1 \mp \gamma_5)/2$ with $q = (u, d)^T, (s, c)^T$.

- Cross section up to $O(1/\Lambda^2)$ is calculated, keeping only the **INTERFERENCE** between the SM and NP.

Amplitude

- **The amplitude for** $q(p_1) + \bar{q}(p_2) \rightarrow t(p_3) + \bar{t}(p_4)$

$$\begin{aligned} |\overline{\mathcal{M}}|^2 \simeq & \frac{4 g_s^4}{9 \hat{s}^2} \left\{ 2m_t^2 \hat{s} \left[1 + \frac{\hat{s}}{2\Lambda^2} (C_1 + C_2) \right] s_{\hat{\theta}}^2 \right. \\ & \left. + \frac{\hat{s}^2}{2} \left[\left(1 + \frac{\hat{s}}{2\Lambda^2} (C_1 + C_2) \right) (1 + c_{\hat{\theta}}^2) + \hat{\beta}_t \left(\frac{\hat{s}}{\Lambda^2} (C_1 - C_2) \right) c_{\hat{\theta}} \right] \right\} \end{aligned}$$

where $C_1 \equiv C_{8q}^{LL} + C_{8q}^{RR}$ and $C_2 \equiv C_{8q}^{LR} + C_{8q}^{RL}$

- $\hat{s} = (p_1 + p_2)^2$, $\hat{\beta}_t^2 = 1 - 4m_t^2/\hat{s}$, and $s_{\hat{\theta}} \equiv \sin \hat{\theta}$ and $c_{\hat{\theta}} \equiv \cos \hat{\theta}$, with $\hat{\theta}$ being the polar angle between the incoming quark and the outgoing top quark in the $t\bar{t}$ rest frame.

- **Neglecting the transverse polarization,**

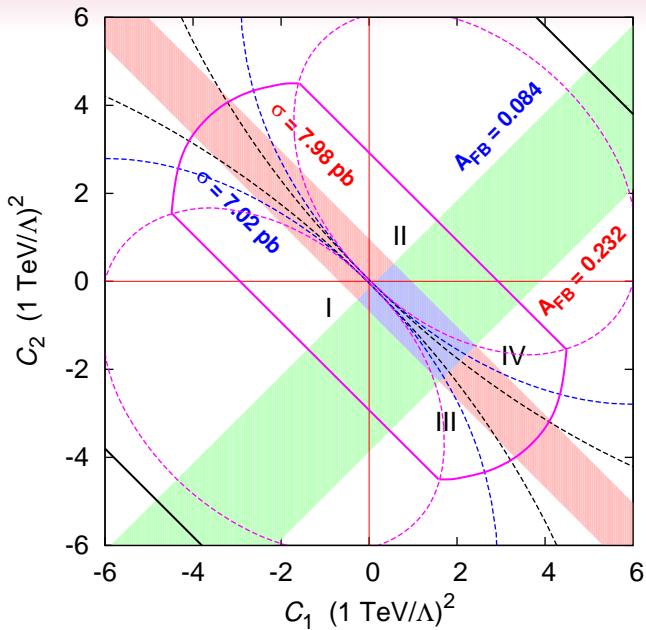
$$\overline{|\mathcal{M}|^2} = \frac{g_s^4}{\hat{s}^2} \left\{ \mathcal{D}_0 + \mathcal{D}_1(P_L + \bar{P}_L) + \mathcal{D}_2(P_L - \bar{P}_L) + \mathcal{D}_3 P_L \bar{P}_L \right\}.$$

The longitudinal polarization of top quark, $P_L \equiv \langle \vec{S}_t \cdot \vec{n}_t \rangle$.

\vec{n}_t is any unit vector defining the spin quantization axis of the top quark.

- **If we choose $\vec{n}_t(\bar{t}) = \vec{p}_t(\bar{t})/|\vec{p}_t(\bar{t})|$ with $\vec{p}_t(\bar{t})$ being the momentum vector of t (\bar{t}), P_L (\bar{P}_L) becomes the usual helicity of (anti)top quark.**

$$\begin{aligned}
 \mathcal{D}_0 &= \frac{2}{9} \cdot \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left(|\langle ++; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 + |\langle --; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 \right. \\
 &\quad \left. + |\langle +-; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 + |\langle -+; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 \right), \\
 \mathcal{D}_1 &= \frac{2}{9} \cdot \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left(|\langle ++; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 - |\langle --; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 \right), \\
 \mathcal{D}_2 &= \frac{2}{9} \cdot \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left(|\langle +-; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 - |\langle -+; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 \right), \\
 \mathcal{D}_3 &= \frac{2}{9} \cdot \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left(|\langle ++; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 + |\langle --; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 \right. \\
 &\quad \left. - |\langle +-; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 - |\langle -+; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 \right),
 \end{aligned}$$



Parity conserving

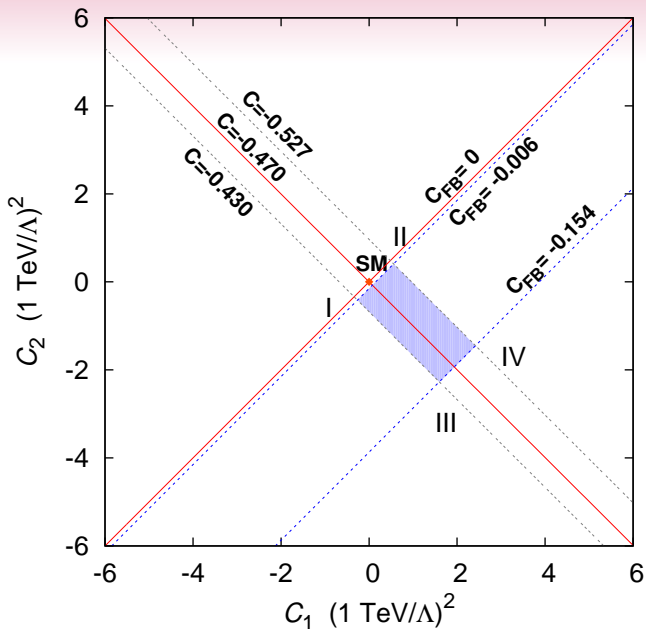
- \mathcal{D}_0 leads to the σ_{tt} and A_{FB} .
- \mathcal{D}_3 leads to the spin-correlation variables,

$$C = \frac{\sigma(t_L \bar{t}_L + t_R \bar{t}_R) - \sigma(t_L \bar{t}_R + t_R \bar{t}_L)}{\sigma(t_L \bar{t}_L + t_R \bar{t}_R) + \sigma(t_L \bar{t}_R + t_R \bar{t}_L)}.$$

- It depends only on the $C_1 + C_2$, so is NOT related to the A_{FB} .
- If we integrate the \mathcal{D}_3 for the forward/backward directions separately, C_{FB} defined as

$$C_{FB} \equiv C(\cos \theta \geq 0) - C(\cos \theta \leq 0),$$

where $C(\cos \theta \geq 0(\leq 0))$ implies the cross sections in the numerator of C are obtained for the forward (backward) region: $\cos \theta \geq 0(\leq 0)$.



Parity Violating

- \mathcal{D}_1 is the coefficient of $P_L + \bar{P}_L$, so it is also CP odd.
- No such terms in EFT analysis, since the heavy particles are integrated out.
- It can arise from the interference between the CP-violating complex couplings together with their decay width etc., if explicit NP form is considered.

Parity violating, continued

- \mathcal{D}_2 is the coefficient of $P_L - \bar{P}_L$, parity odd but CP-even.
- Explicitly,

$$\mathcal{D}_2 \simeq \frac{\hat{s}}{9\Lambda^2} \left[(C'_1 + C'_2)\hat{\beta}_t(1 + c_\theta^2) + (C'_1 - C'_2)(5 - 3\hat{\beta}_t^2)c_\theta \right]$$

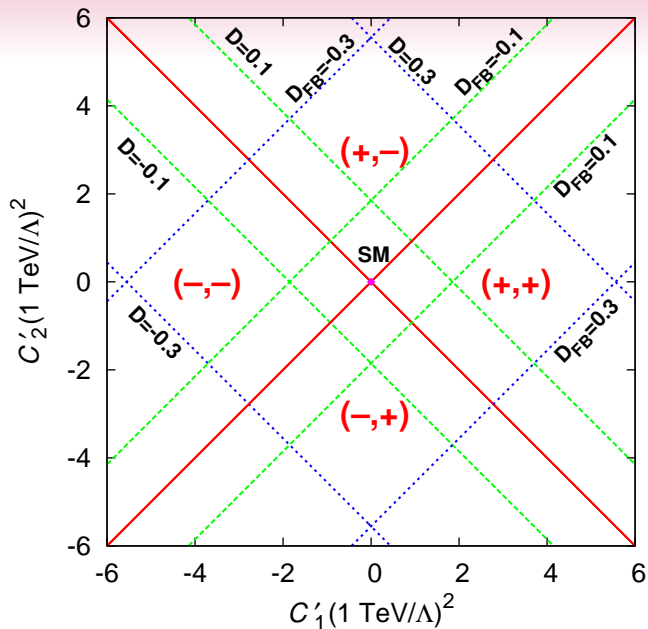
with

$$C'_1 \equiv C_{8q}^{RR} - C_{8q}^{LL}, \quad C'_2 \equiv C_{8q}^{LR} - C_{8q}^{RL}.$$

- As σ_{tt} , A_{FB} and C , C_{FB} , new two polarization observables can be drawn.

$$D \equiv \frac{\sigma(t_R\bar{t}_L) - \sigma(t_L\bar{t}_R)}{\sigma(t_R\bar{t}_R) + \sigma(t_L\bar{t}_L) + \sigma(t_L\bar{t}_R) + \sigma(t_R\bar{t}_L)},$$

$$D_{FB} \equiv D(\cos\hat{\theta} \geq 0) - D(\cos\hat{\theta} \leq 0)$$



Model independent vs. Models

- We can draw some information on each models from model-independent study.
- Integrating out heavy fields, the Wilson coefficients are,

$$\frac{C_{8q}^{RR}}{\Lambda^2} = -\frac{g_{8q}^R g_{8t}^R}{m_{V_{8R}}^2} - \frac{2|\tilde{g}_{1q}^R|^2}{m_{\tilde{V}_{1R}}^2} + \frac{1}{N_C} \frac{|\tilde{g}_{8q}^R|^2}{m_{V_{8R}}^2},$$

$$\frac{C_{8q}^{LL}}{\Lambda^2} = -\frac{g_{8q}^L g_{8t}^L}{m_{V_{8L}}^2} - \frac{2|\tilde{g}_{1q}^L|^2}{m_{\tilde{V}_{1L}}^2} + \frac{1}{N_C} \frac{|\tilde{g}_{8q}^L|^2}{m_{V_{8L}}^2},$$

$$\frac{C_{8q}^{LR}}{\Lambda^2} = -\frac{g_{8q}^L g_{8t}^R}{m_{V_8}^2} - \frac{|\tilde{\eta}_{1q}^L|^2}{m_{\tilde{S}_{1L}}^2} + \frac{1}{2N_C} \frac{|\tilde{\eta}_{8q}^L|^2}{m_{\tilde{S}_{8L}}^2},$$

$$\frac{C_{8q}^{RL}}{\Lambda^2} = -\frac{g_{8q}^R g_{8t}^L}{m_{V_8}^2} - \frac{|\tilde{\eta}_{1q}^R|^2}{m_{\tilde{S}_{1R}}^2} + \frac{1}{2N_C} \frac{|\tilde{\eta}_{8q}^R|^2}{m_{\tilde{S}_{8R}}^2},$$

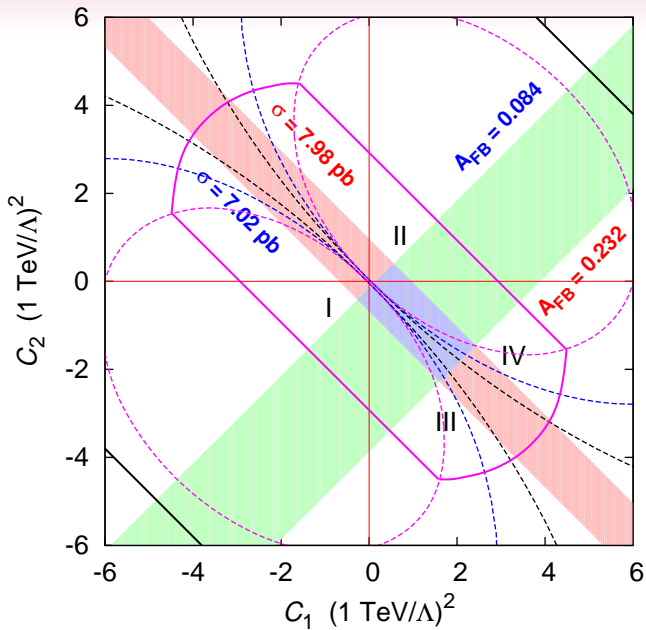
- **Another interesting possibility is**
 - color-triplet S_k^γ with mass m_{S_3} and
 - color-sextet $S_{ij}^{\alpha\beta}$ with mass m_{S_6} with the SM quarks .
- **With the following interactions**

$$\mathcal{L} = g_s \left[\frac{\eta_3}{2} \epsilon_{\alpha\beta\gamma} \epsilon^{ijk} u_{iR}^\alpha u_{jR}^\beta S_k^\gamma + \eta_6 u_{iR}^\alpha u_{jR}^\beta S_{ij}^{\alpha\beta} + h.c. \right]$$

the u -channel exchange of new scalars can contribute to $u\bar{u} \rightarrow t\bar{t}$, resulting in

$$\frac{C_{8u}^{RR}}{\Lambda^2} = -\frac{|\eta_3|^2}{m_{S_3}^2} + \frac{2|\eta_6|^2}{m_{S_6}^2}.$$

Resonance	C^{RR}	C^{LL}	C^{LR}	C^{RL}	$C_1 - C_2$	$C'_1 + C'_2$	$C'_1 - C'_2$	A_{FB}
\tilde{V}_{1R}	-	0	0	0	-	-	-	×
\tilde{V}_{1L}	0	-	0	0	-	+	+	×
\tilde{V}_{8R}	+	0	0	0	+	+	+	✓
\tilde{V}_{8L}	0	+	0	0	+	-	-	✓
\tilde{S}_{1R}	0	0	0	-	+	+	-	✓
\tilde{S}_{1L}	0	0	-	0	+	-	+	✓
\tilde{S}_{8R}	0	0	0	+	-	-	+	×
\tilde{S}_{8L}	0	0	+	0	-	+	-	×
S_2^α	-	0	0	0	-	-	-	×
$S_{13}^{\alpha\beta}$	+	0	0	0	+	+	+	✓
V_{8R}	±	0	0	0	±	±	±	√(+) or ×(-)
V_{8L}	0	±	0	0	±	∓	∓	√(+) or ×(-)
V_{8R}, V_{8L}	indef.	indef.	indef.	indef.	indef.	indef.	indef.	indef.



Simple cases

- Following four cases are simple :

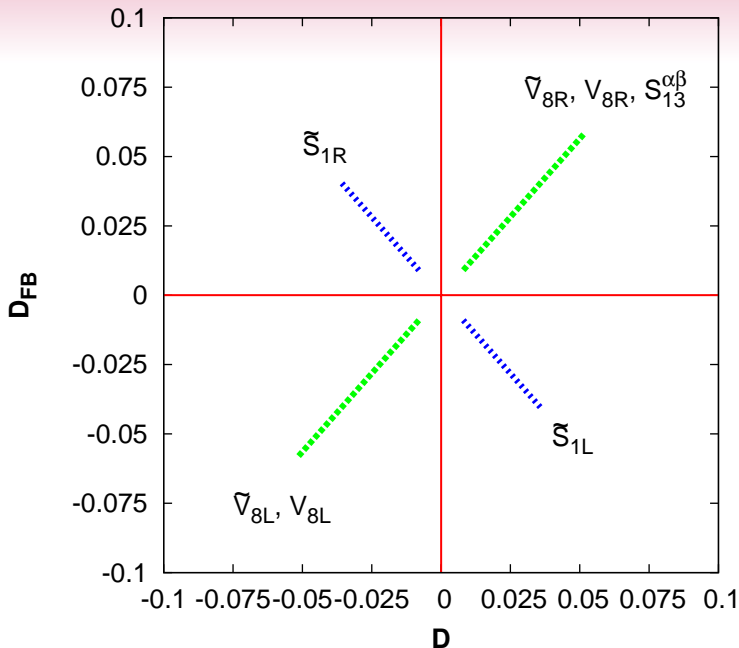
$$V_{8R,8L} : -\frac{g_{8q}^{R,L} g_{8t}^{R,L}}{m_{V_{8R,L}}^2} \simeq 0.649 \pm 0.024, ,$$

$$\tilde{V}_{8R,8L} : \frac{1}{N_c} \left(\frac{1 \text{ TeV}}{m_{\tilde{V}_{8R,8L}}} \right)^2 |\tilde{g}_{8q}^{R,L}|^2 \simeq 0.649 \pm 0.024, ,$$

$$\tilde{S}_{1R,1L} : \left(\frac{1 \text{ TeV}}{m_{\tilde{S}_{1R,1L}}} \right)^2 |\tilde{\eta}_{1q}^{R,L}|^2 \simeq 0.799 \pm 0.174, ,$$

$$S_{13}^{\alpha\beta} : 2 \left(\frac{1 \text{ TeV}}{m_{S_6}} \right)^2 |\eta_6|^2 \simeq 0.649 \pm 0.024 .$$

- New spin-correlation observables D and D_{FB} values are predicted for each case.



More general case

- In general, V_{8L} and V_{8R} can coexist. In that case the new observables are not so definite compared to the previous cases.
- We introduce new parameterizations

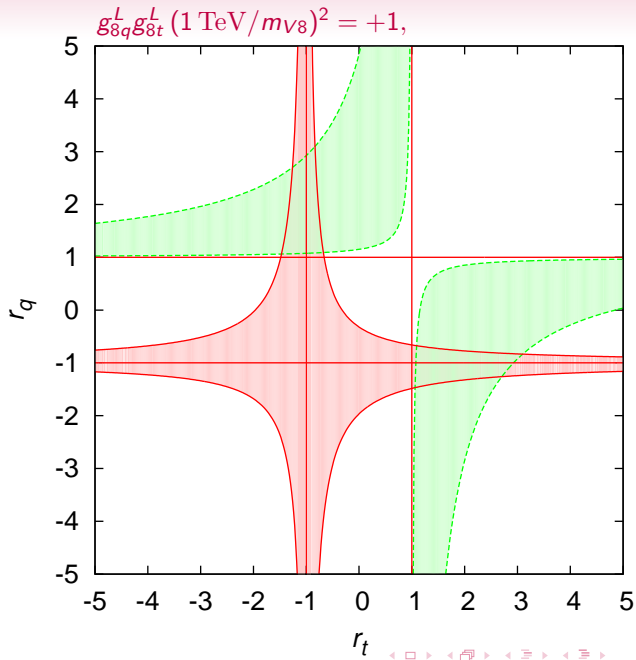
$$(C_1 + C_2)/\Lambda^2 = -g_{8q}^L g_{8t}^L (r_q + 1)(r_t + 1)/m_{V8}^2$$

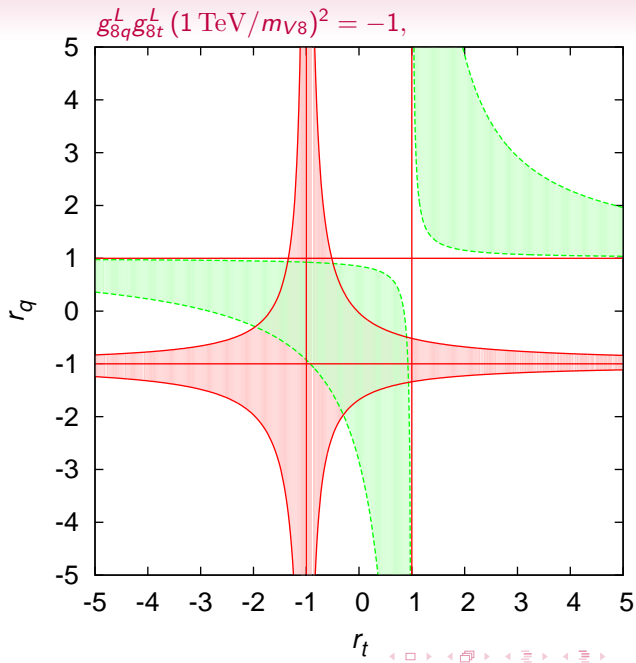
$$(C_1 - C_2)/\Lambda^2 = -g_{8q}^L g_{8t}^L (r_q - 1)(r_t - 1)/m_{V8}^2$$

$$(C'_1 + C'_2)/\Lambda^2 = -g_{8q}^L g_{8t}^L (r_q + 1)(r_t - 1)/m_{V8}^2$$

$$(C'_1 - C'_2)/\Lambda^2 = -g_{8q}^L g_{8t}^L (r_q - 1)(r_t + 1)/m_{V8}^2$$

with $r_q \equiv g_{8q}^R/g_{8q}^L$ and $r_t \equiv g_{8t}^R/g_{8t}^L$. Any deviation of r_q (r_t) from 1 characterizes P violation in the light (top) quark sector.





Important observations are

- **By observing the relation**

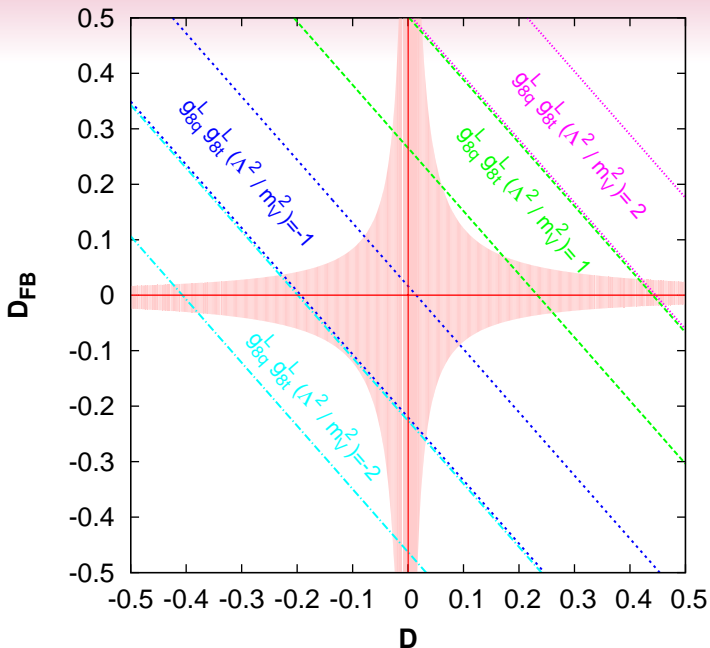
$(C_1 + C_2)(C_1 - C_2) = (C'_1 + C'_2)(C'_1 - C'_2)$ **which leads to**

$$\Delta\sigma_{t\bar{t}} \Delta A_{\text{FB}} \propto D D_{\text{FB}}.$$

Let us note that $\Delta\sigma_{t\bar{t}} \propto (C_1 + C_2)$, $\Delta A_{\text{FB}} \propto (C_1 + C_2)$,
 $D \propto (C'_1 + C'_2)$, **and** $D_{\text{FB}} \propto (C'_1 - C'_2)$.

- **Furthermore, we observe**

$$\begin{aligned} & g_{8q}^L g_{8t}^L \left(\frac{\Lambda}{m_{V8}} \right)^2 \\ = & \frac{[(C'_1 + C'_2) - (C_1 + C_2)] [(C'_1 + C'_2) - (C_1 - C_2)]}{4(C'_1 + C'_2)} \\ = & \frac{1}{4} [(C'_1 + C'_2) - (C_1 + C_2) - (C_1 - C_2) + (C'_1 - C'_2)] , \end{aligned}$$



Summary and Prospect

- **LHC is a kind of top factory.**
- **NP is highly probable to appear in the top pair production channel.**
- **Polarization observables are very useful for discriminating many forms of the NP models.**
- **Discrete symmetries such as P,CP etc., can be analysed also in this helicity amplitude setup.**
- **Complete and general analysis with FULL theory is in progress, including above 16 observables (\mathcal{D}_1 and more) in relation with the Tevatron/LHC signals.**