The Forward-Backward Asymmetry and Polarization Observables

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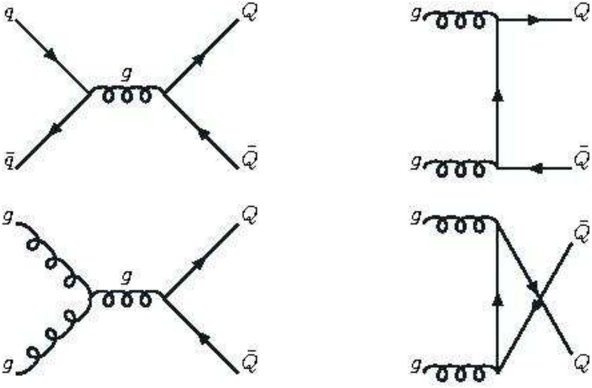
Based on
Phys.Lett.B691:238-242,2010,
arXiv:1012.0102,
arXiv:1104.4443,
and more in progress

with P. Ko (KIAS), J. S. Lee (NCTS) and S. Nam (KISTI)

May 24, 2011 @ KIAS

Tevatron with $\sqrt{S} = 1.96$ TeV,

- $\sigma_{t\bar{t}} = 7.50 \pm 0.48$ pb
- $q\bar{q} \to t\bar{t} \sim 85\%$
- $gg \to t\bar{t} \sim 15\%$



Foward-backward (FB) asymmetry @Tevatron [1]

- **In the laboratory frame:**

$$A_{\rm FB}^{p\bar{p}} = 0.150 \pm 0.055 \ (\rm stat + sys)$$

- **In the $t\bar{t}$ rest frame (parton level):**

$$A_{\rm FB}^{t\bar{t}} = 0.158 \pm 0.075 \ (\rm stat + sys) \ [0.058 \pm 0.009]$$

- **In the $t\bar{t}$ rest frame (fully corrected):**

$$A_{\rm FB}^{t\bar{t}}(|\Delta y| < 1.0) = 0.026 \pm 0.118 \ [0.039 \pm 0.006]$$

$$A_{\rm FB}^{t\bar{t}}(|\Delta y| \geq 1.0) = 0.611 \pm 0.256 \ [0.123 \pm 0.008]$$

- **In the $t\bar{t}$ rest frame (parton level):**

$$A_{\rm FB}^{t\bar{t}}(M_{t\bar{t}} < 450 \ \rm GeV) = -0.116 \pm 0.153 \ [0.040 \pm 0.006]$$

$$A_{\rm FB}^{t\bar{t}}(M_{t\bar{t}} \geq 450 \ \rm GeV) = 0.475 \pm 0.114 \ [0.088 \pm 0.013]$$

[1] The values in the squared bracket are SM predictions with MCFM

Systematic study of New Physics (NP)

With $q = u, d, s, c, b$**,**

- **Spin-1 NP :**

$$
\begin{aligned}
\mathcal{L}_{\rm int}^{\rm spin-1} &= g_s \sum_{A=L,R} V_{1A}^{\mu} \left[g_{1q}^{A} \, (\bar{q}_A \gamma_\mu q_A) + g_{1t}^{A} \, (\bar{t}_A \gamma_\mu t_A) \right] \\
&+ g_s \sum_{A=L,R} V_{8A}^{a\mu} \left[g_{8q}^{A} \, (\bar{q}_A \gamma_\mu T^a q_A) + g_{8t}^{A} \, (\bar{t}_A \gamma_\mu T^a t_A) \right] \\
&+ g_s \sum_{A=L,R} \left[\tilde{V}_{1A}^{\mu} \, \tilde{g}_{1q}^{A} \, (\bar{t}_A \gamma_\mu q_A) + \tilde{V}_{8A}^{a\mu} \, \tilde{g}_{8q}^{A} \, (\bar{t}_A \gamma_\mu T^a q_A) + \mathrm{h.c.} \right]
\end{aligned}
$$

Systematic study of New Physics (NP), continued

- **Spin-0 NP :**

$$
\begin{aligned}
\mathcal{L}_{\rm int}^{\rm spin-0} &= g_s \sum_{A=L,R} S_{1A} \left[\eta_{1q}^{A} \, (\bar{q}\, P_A\, q) + \eta_{1t}^{A} \, (\bar{t}\, P_A\, t) \right] \\
&+ g_s \sum_{A=L,R} S_{8A}^{a} \left[\eta_{8q}^{A} \, (\bar{q}\, T^a P_A\, q) + \eta_{8t}^{A} \, (\bar{t}\, T^a P_A\, t) \right] \\
&+ g_s \sum_{A=L,R} \left[\tilde{S}_{1A} \, \tilde{\eta}_{1q}^{A} \, (\bar{t}\, P_A\, q) + \tilde{S}_{8A}^{a} \, \tilde{\eta}_{8q}^{A} \, (\bar{t}\, T^a P_A\, q) + \text{h.c.} \right] \\
&+ g_s \, \epsilon_{\alpha\beta\gamma} \left[\tilde{\eta}_{3q} \left(q_R^{\alpha} \; t_R^{\beta} \, \tilde{S}_3^{\gamma} \right) + \text{h.c.} \right] \\
&+ g_s \left[\tilde{\eta}_{6q} \left(q_R^{\alpha} \; t_R^{\beta} \, \tilde{S}_6^{\alpha\beta} \right) + \text{h.c.} \right]
\end{aligned}
$$

- **Most generally, the amplitude for the process $q\,\bar{q} \to t\,\bar{t}$ can be cast into the form**

$$
\begin{aligned}
&\mathcal{M}(q\bar{q} \to t\bar{t}) \\
&\quad = \frac{g_s^2}{\hat{s}} \sum_{A,B=L,R} \Big\{ C_{S1}^{AB} \, [\overline{v_q(k_2)}\, P_A\, u_q(k_1)]\; [\overline{u_t(p_1)}\, P_B\, v_t(p_2)] \\
&\qquad + C_{S8}^{AB} \, [\overline{v_q(k_2)}\, P_A\, T^a\, u_q(k_1)]\; [\overline{u_t(p_1)}\, P_B\, T^a\, v_t(p_2)] \\
&\qquad + C_{V1}^{AB} \, [\overline{v_q(k_2)}\, \gamma_\mu\, P_A\, u_q(k_1)]\; [\overline{u_t(p_1)}\, \gamma^\mu\, P_B\, v_t(p_2)] \\
&\qquad + C_{V8}^{AB} \, [\overline{v_q(k_2)}\, \gamma_\mu\, P_A\, T^a\, u_q(k_1)]\; [\overline{u_t(p_1)}\, \gamma^\mu\, P_B\, T^a\, v_t(p_2)] \\
&\qquad + C_{T1}^{A} \, [\overline{v_q(k_2)}\, \sigma_{\mu\nu}\, P_A\, u_q(k_1)]\; [\overline{u_t(p_1)}\, \sigma^{\mu\nu}\, P_A\, v_t(p_2)] \\
&\qquad + C_{T8}^{A} \, [\overline{v_q(k_2)}\, \sigma_{\mu\nu}\, P_A\, T^a\, u_q(k_1)]\; [\overline{u_t(p_1)}\, \sigma^{\mu\nu}\, P_A\, T^a\, v_t(p_2)] \Big\}
\end{aligned}
$$

where $P_A = (1 + \alpha\gamma_5)/2$ with $\alpha = -\ (+)$ for $A = L\ (R)$.

Table: $\tilde{x} \equiv \hat{s}/(\hat{x} - M^2 + iM\Gamma)$ with $x = s, t, u$.

Coef.	SM	V_1^μ	$V_8^{a\mu}$	$\tilde{V}_1^\mu$	$\tilde{V}_8^{a\mu}$
C_{S1}^{LL}					
C_{S1}^{RR}					
C_{S1}^{LR}				$-\frac{2}{N}\tilde{g}_{1q}^L(\tilde{g}_{1q}^R)^*\,\tilde{t}$	$-\left(1-\frac{1}{N^2}\right)\tilde{g}_{8q}^L(\tilde{g}_{8q}^R)^*\,\tilde{t}$
C_{S1}^{RL}				$-\frac{2}{N}\tilde{g}_{1q}^R(\tilde{g}_{1q}^L)^*\,\tilde{t}$	$-\left(1-\frac{1}{N^2}\right)\tilde{g}_{8q}^R(\tilde{g}_{8q}^L)^*\,\tilde{t}$
C_{S8}^{LL}					
C_{S8}^{RR}					
C_{S8}^{LR}				$-4\tilde{g}_{1q}^L(\tilde{g}_{1q}^R)^*\,\tilde{t}$	$\frac{2}{N}\tilde{g}_{8q}^L(\tilde{g}_{8q}^R)^*\,\tilde{t}$
C_{S8}^{RL}				$-4\tilde{g}_{1q}^R(\tilde{g}_{1q}^L)^*\,\tilde{t}$	$\frac{2}{N}\tilde{g}_{8q}^R(\tilde{g}_{8q}^L)^*\,\tilde{t}$
C_{V1}^{LL}		$g_{1q}^L g_{1t}^L\,\tilde{s}$		$\frac{1}{N}\lvert\tilde{g}_{1q}^L\rvert^2\,\tilde{t}$	$\frac{1}{2}\left(1-\frac{1}{N^2}\right)\lvert\tilde{g}_{8q}^L\rvert^2\,\tilde{t}$
C_{V1}^{RR}		$g_{1q}^R g_{1t}^R\,\tilde{s}$		$\frac{1}{N}\lvert\tilde{g}_{1q}^R\rvert^2\,\tilde{t}$	$\frac{1}{2}\left(1-\frac{1}{N^2}\right)\lvert\tilde{g}_{8q}^R\rvert^2\,\tilde{t}$
C_{V1}^{LR}		$g_{1q}^L g_{1t}^R\,\tilde{s}$			
C_{V1}^{RL}		$g_{1q}^R g_{1t}^L\,\tilde{s}$			
C_{V8}^{LL}	1		$g_{8q}^L g_{8t}^L\,\tilde{s}$	$2\lvert\tilde{g}_{1q}^L\rvert^2\,\tilde{t}$	$-\frac{1}{N}\lvert\tilde{g}_{8q}^L\rvert^2\,\tilde{t}$
C_{V8}^{RR}	1		$g_{8q}^R g_{8t}^R\,\tilde{s}$	$2\lvert\tilde{g}_{1q}^R\rvert^2\,\tilde{t}$	$-\frac{1}{N}\lvert\tilde{g}_{8q}^R\rvert^2\,\tilde{t}$
C_{V8}^{LR}	1		$g_{8q}^L g_{8t}^R\,\tilde{s}$		
C_{V8}^{RL}	1		$g_{8q}^R g_{8t}^L\,\tilde{s}$		
C_{T1}^{L}					
C_{T1}^{R}					
C_{T8}^{L}					
C_{T8}^{R}					

Coef.	S_1	S_8^a	$\tilde{S}_1$	$\tilde{S}_8^a$	$\tilde{S}_3^\gamma$	$\tilde{S}_6^{\alpha\beta}$
C_{S1}^{LL}	$-\eta_{1q}^L\eta_{1t}^L\,\tilde{s}$		$\frac{1}{2N}\tilde{\eta}_{1q}^L(\tilde{\eta}_{1q}^R)^*\,\tilde{t}$	$\frac{1}{4}\left(1-\frac{1}{N^2}\right)\tilde{\eta}_{8q}^L(\tilde{\eta}_{8q}^R)^*\,\tilde{t}$		
C_{S1}^{RR}	$-\eta_{1q}^R\eta_{1t}^R\,\tilde{s}$		$\frac{1}{2N}\tilde{\eta}_{1q}^R(\tilde{\eta}_{1q}^L)^*\,\tilde{t}$	$\frac{1}{4}\left(1-\frac{1}{N^2}\right)\tilde{\eta}_{8q}^R(\tilde{\eta}_{8q}^L)^*\,\tilde{t}$		
C_{S1}^{LR}	$-\eta_{1q}^L\eta_{1t}^R\,\tilde{s}$					
C_{S1}^{RL}	$-\eta_{1q}^R\eta_{1t}^L\,\tilde{s}$					
C_{S8}^{LL}		$-\eta_{8q}^L\eta_{8t}^L\,\tilde{s}$	$\tilde{\eta}_{1q}^L(\tilde{\eta}_{1q}^R)^*\,\tilde{t}$	$-\frac{1}{2N}\tilde{\eta}_{8q}^L(\tilde{\eta}_{8q}^R)^*\,\tilde{t}$		
C_{S8}^{RR}		$-\eta_{8q}^R\eta_{8t}^R\,\tilde{s}$	$\tilde{\eta}_{1q}^R(\tilde{\eta}_{1q}^L)^*\,\tilde{t}$	$-\frac{1}{2N}\tilde{\eta}_{8q}^L(\tilde{\eta}_{8q}^R)^*\,\tilde{t}$		
C_{S8}^{LR}		$-\eta_{8q}^L\eta_{8t}^R\,\tilde{s}$				
C_{S8}^{RL}		$-\eta_{8q}^R\eta_{8t}^L\,\tilde{s}$				
C_{V1}^{LL}						
C_{V1}^{RR}					$-\frac{1}{2}\left(1-\frac{1}{N}\right)\lvert\tilde{\eta}_{3q}\rvert^2\,\tilde{u}$	$-\frac{1}{4}\left(1+\frac{1}{N}\right)\lvert\tilde{\eta}_{6q}\rvert^2\,\tilde{u}$
C_{V1}^{LR}			$\frac{1}{2N}\lvert\tilde{\eta}_{1q}^L\rvert^2\,\tilde{t}$	$\frac{1}{4}\left(1-\frac{1}{N^2}\right)\lvert\tilde{\eta}_{8q}^L\rvert^2\,\tilde{t}$		
C_{V1}^{RL}			$\frac{1}{2N}\lvert\tilde{\eta}_{1q}^R\rvert^2\,\tilde{t}$	$\frac{1}{4}\left(1-\frac{1}{N^2}\right)\lvert\tilde{\eta}_{8q}^R\rvert^2\,\tilde{t}$		
C_{V8}^{LL}						
C_{V8}^{RR}					$\lvert\tilde{\eta}_{3q}\rvert^2\,\tilde{u}$	$-\frac{1}{2}\lvert\tilde{\eta}_{6q}\rvert^2\,\tilde{u}$
C_{V8}^{LR}			$\lvert\tilde{\eta}_{1q}^L\rvert^2\,\tilde{t}$	$-\frac{1}{2N}\lvert\tilde{\eta}_{8q}^L\rvert^2\,\tilde{t}$		
C_{V8}^{RL}			$\lvert\tilde{\eta}_{1q}^R\rvert^2\,\tilde{t}$	$-\frac{1}{2N}\lvert\tilde{\eta}_{8q}^R\rvert^2\,\tilde{t}$		
C_{T1}^{L}			$\frac{1}{8N}\tilde{\eta}_{1q}^L(\tilde{\eta}_{1q}^R)^*\,\tilde{t}$	$\frac{1}{16}\left(1-\frac{1}{N^2}\right)\tilde{\eta}_{8q}^L(\tilde{\eta}_{8q}^R)^*\,\tilde{t}$		
C_{T1}^{R}			$\frac{1}{8N}\tilde{\eta}_{1q}^R(\tilde{\eta}_{1q}^L)^*\,\tilde{t}$	$\frac{1}{16}\left(1-\frac{1}{N^2}\right)\tilde{\eta}_{8q}^R(\tilde{\eta}_{8q}^L)^*\,\tilde{t}$		
C_{T8}^{L}			$\frac{1}{4}\tilde{\eta}_{1q}^L(\tilde{\eta}_{1q}^R)^*\,\tilde{t}$	$-\frac{1}{8N}\tilde{\eta}_{8q}^L(\tilde{\eta}_{8q}^R)^*\,\tilde{t}$		
C_{T8}^{R}			$\frac{1}{4}\tilde{\eta}_{1q}^R(\tilde{\eta}_{1q}^L)^*\,\tilde{t}$	$-\frac{1}{8N}\tilde{\eta}_{8q}^R(\tilde{\eta}_{8q}^L)^*\,\tilde{t}$		

Effective Field Theory (EFT) approach

- **If the NP particles are heavy enough, we can adopt the veiwpoint of effective Lagragian.**
- **Assuming $SU(2)_L \times U(1)_Y$ and the custodial symmetry $SU(2)_R$ for the light quark sector,**

$$\mathcal{L}_6 = \frac{g_s^2}{\Lambda^2} \sum_{A,B} \Big[C_{1q}^{AB} (\bar{q}_A \gamma_\mu q_A)(\bar{t}_B \gamma^\mu t_B) + C_{8q}^{AB} (\bar{q}_A T^a \gamma_\mu q_A)(\bar{t}_B T^a \gamma^\mu t_B) \Big]$$

where $T^a = \lambda^a/2$, $\{A, B\} = \{L, R\}$, **and** $L, R \equiv (1 \mp \gamma_5)/2$ **with** $q = (u, d)^T, (s, c)^T$.

- **Cross section up to $O(1/\Lambda^2)$ is calculated, keeping only the INTERFERENCE between the SM and NP.**

Amplitude

- **The amplitude for** $q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4)$

$$\overline{|\mathcal{M}|^2} \simeq \frac{4\, g_s^4}{9\, \hat{s}^2} \left\{ 2m_t^2 \hat{s} \left[1 + \frac{\hat{s}}{2\Lambda^2} (C_1 + C_2) \right] s_{\hat{\theta}}^2 \right.$$
$$\left. + \frac{\hat{s}^2}{2} \left[\left(1 + \frac{\hat{s}}{2\Lambda^2} (C_1 + C_2) \right) (1 + c_{\hat{\theta}}^2) + \hat{\beta}_t \left(\frac{\hat{s}}{\Lambda^2} (C_1 - C_2) \right) c_{\hat{\theta}} \right] \right\}$$

where $C_1 \equiv C_{8q}^{LL} + C_{8q}^{RR}$ **and** $C_2 \equiv C_{8q}^{LR} + C_{8q}^{RL}$

- $\hat{s} = (p_1 + p_2)^2$, $\hat{\beta}_t^2 = 1 - 4m_t^2/\hat{s}$, **and** $s_{\hat{\theta}} \equiv \sin\hat{\theta}$ **and** $c_{\hat{\theta}} \equiv \cos\hat{\theta}$, **with** $\hat{\theta}$ **being the polar angle between the incoming quark and the outgoing top quark in the** $t\bar{t}$ **rest frame.**

Results

- **Convoluted with CTEQ6L and K factor 1.3.**

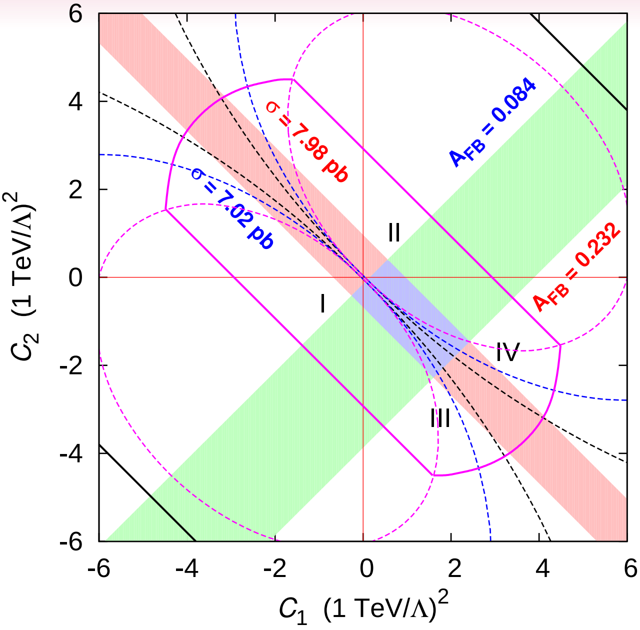
- **For validity check, we impose three criteria,**

$$\sigma = \sigma_{SM} + \sigma_{int} + \sigma_{NP},$$

$$\begin{aligned} |\sigma_{int}| &< r \times \sigma_{SM} \quad (straight) \\ \sigma_{NP} &< r \times |\sigma_{int}| \quad (two\ ellipses\ adjacent\ at\ the\ origin) \\ \sigma_{NP} &< r^2 \times \sigma_{SM} \quad (ellipses\ centered\ at\ the\ origin) \end{aligned}$$

with $r = 0.3,\ 0.5\ 1.0.$

- $\Delta\sigma_{t\bar{t}} \equiv \sigma_{t\bar{t}} - \sigma_{t\bar{t}}^{\mathrm{SM}} \propto (C_1 + C_2)$,
  $\Delta A_{\mathrm{FB}} \equiv A_{\mathrm{FB}} - A_{\mathrm{FB}}^{\mathrm{SM}} \propto (C_1 - C_2)$.



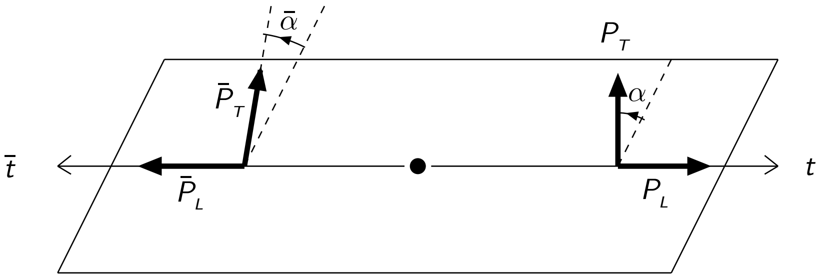
σ = 7.98 pb
σ = 7.02 pb
A_FB = 0.084
A_FB = 0.232
I
II
III
IV
6
4
2
0
-2
-4
-6
C_2 (1 TeV/Λ)^2
C_1 (1 TeV/Λ)^2

Polarization Observables

- **In order to have nonzero NP contribution to $A_{\rm FB}$, we need $C_1 - C_2 \neq 0$.**
- $C_1 - C_2 = C_{8q}^{LL} + C_{8q}^{RR} - C_{8q}^{LR} - C_{8q}^{RL}$

  $\rightarrow$ **Parity violation on both sectors, top and light quark.**
- $\sigma_{t\bar{t}}$ **and** $A_{\rm FB}$ **depend only on two combinations** C_1 **and** C_2

  $\rightarrow$ **Not enough to draw the information on chiral structure of NP.**
- **More observables are required.**



The $t\bar{t}$ production plane in its rest frame.
The longitudinal-polarization vector $P_L(\bar{P}_L)$ and the transverse-polarization vector $P_T(\bar{P}_T)$ with the azimuthal angle $\alpha(\bar{\alpha})$ of $t\,(\bar{t})$ are shown.

- **The amplitude squared can be spanned as**

$$
\begin{aligned}
\overline{|\mathcal{M}|^2} \;=\; \frac{g_s^4}{\hat{s}^2}\Big\{ & D_0 + D_1(P_L + \bar{P}_L) + D_2(P_L - \bar{P}_L) + D_3 P_L \bar{P}_L \\
& + [D_4 \sin(\alpha - \bar{\alpha}) + D_5 \cos(\alpha - \bar{\alpha}) + D_6 \sin(\alpha + \bar{\alpha}) + D_7 \cos(\alpha + \bar{\alpha})] P_T \bar{P}_T \\
& + D_8(P_T \sin\alpha + \bar{P}_T \sin\bar{\alpha}) + D_9(P_T \sin\alpha - \bar{P}_T \sin\bar{\alpha}) \\
& + D_{10}(P_T \cos\alpha + \bar{P}_T \cos\bar{\alpha}) + D_{11}(P_T \cos\alpha - \bar{P}_T \cos\bar{\alpha}) \\
& + D_{12}(P_L \bar{P}_T \sin\bar{\alpha} + \bar{P}_L P_T \sin\alpha)) + D_{13}(P_L \bar{P}_T \sin\bar{\alpha} - \bar{P}_L P_T \sin\alpha) \\
& + D_{14}(P_L \bar{P}_T \cos\bar{\alpha} + \bar{P}_L P_T \cos\alpha)) + D_{15}(P_L \bar{P}_T \cos\bar{\alpha} - \bar{P}_L P_T \cos\alpha) \Big\}
\end{aligned}
$$

- **The coefficients D_i's are expressed by helicity amplitudes**

$$
\begin{aligned}
D_0 &= \frac{1}{4}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(++;\lambda\bar\lambda)|^2 + |\mathcal{A}(--;\lambda\bar\lambda)|^2 + |\mathcal{A}(+-;\lambda\bar\lambda)|^2 + |\mathcal{A}(-+;\lambda\bar\lambda)|^2\right),\\
D_1 &= \frac{1}{4}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(++;\lambda\bar\lambda)|^2 - |\mathcal{A}(--;\lambda\bar\lambda)|^2\right),\\
D_2 &= \frac{1}{4}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(+-;\lambda\bar\lambda)|^2 - |\mathcal{A}(-+;\lambda\bar\lambda)|^2\right),\\
D_3 &= \frac{1}{4}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}} \left(|\mathcal{A}(++;\lambda\bar\lambda)|^2 + |\mathcal{A}(--;\lambda\bar\lambda)|^2 - |\mathcal{A}(+-;\lambda\bar\lambda)|^2 - |\mathcal{A}(-+;\lambda\bar\lambda)|^2\right),\\
D_4 &= \frac{1}{2}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}}\, \Im\mathrm{m}[\mathcal{A}(++;\lambda\bar\lambda)\,\mathcal{A}^*(--;\lambda\bar\lambda)],\\
D_5 &= -\frac{1}{2}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}}\, \Re\mathrm{e}[\mathcal{A}(++;\lambda\bar\lambda)\,\mathcal{A}^*(--;\lambda\bar\lambda)],\\
D_6 &= \frac{1}{2}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}}\, \Im\mathrm{m}[\mathcal{A}(+-;\lambda\bar\lambda)\,\mathcal{A}^*(-+;\lambda\bar\lambda)^*],\\
& \cdots\\
& \cdots\\
D_{15} &= -\frac{1}{4}\sum_{\lambda,\bar\lambda,\mathcal{A}} f_{\mathcal{A}} \Big(\Re\mathrm{e}[\mathcal{A}(++;\lambda\bar\lambda)\,\mathcal{A}^*(+-;\lambda\bar\lambda)] - \Re\mathrm{e}[\mathcal{A}(--;\lambda\bar\lambda)\,\mathcal{A}^*(-+;\lambda\bar\lambda)]\\
&\qquad - \Re\mathrm{e}[\mathcal{A}(--;\lambda\bar\lambda)\,\mathcal{A}^*(+-;\lambda\bar\lambda)] + \Re\mathrm{e}[\mathcal{A}(++;\lambda\bar\lambda)\,\mathcal{A}^*(-+;\lambda\bar\lambda)]\Big),
\end{aligned}
$$

- **Neglecting the transverse polarization,**

$$\overline{|\mathcal{M}|^2} = \frac{g_s^4}{\hat{s}^2}\left\{\mathcal{D}_0 + \mathcal{D}_1(P_L + \bar{P}_L) + \mathcal{D}_2(P_L - \bar{P}_L) + \mathcal{D}_3 P_L \bar{P}_L\right\}.$$

• **The longitudinal polarization of top quark,**
$P_L \equiv \langle \vec{S}_t \cdot \vec{n}_t \rangle$.
• $\vec{n}_t$ **is any unit vector defining the spin quantization axis of the top quark,**

- **If we choose** $\vec{n}_{t\,(\bar{t})} = \vec{p}_{t\,(\bar{t})}/|\vec{p}_{t\,(\bar{t})}|$ **with** $\vec{p}_{t\,(\bar{t})}$ **being the momentum vector of** t **(**$\bar{t}$**),** P_L **(**$\bar{P}_L$**) becomes the usual helicity of (anti)top quark.**
  $\rightarrow$ **HELICITY BASIS**

$$\mathcal{D}_0 = \frac{2}{9}\cdot\frac{1}{4}\sum_{\lambda,\bar\lambda}\Big(|\langle ++;\lambda\bar\lambda\rangle_{\rm oct}|^2 + |\langle --;\lambda\bar\lambda\rangle_{\rm oct}|^2 + |\langle +-;\lambda\bar\lambda\rangle_{\rm oct}|^2 + |\langle -+;\lambda\bar\lambda\rangle_{\rm oct}|^2\Big),$$

$$\mathcal{D}_1 = \frac{2}{9}\cdot\frac{1}{4}\sum_{\lambda,\bar\lambda}\Big(|\langle ++;\lambda\bar\lambda\rangle_{\rm oct}|^2 - |\langle --;\lambda\bar\lambda\rangle_{\rm oct}|^2\Big),$$

$$\mathcal{D}_2 = \frac{2}{9}\cdot\frac{1}{4}\sum_{\lambda,\bar\lambda}\Big(|\langle +-;\lambda\bar\lambda\rangle_{\rm oct}|^2 - |\langle -+;\lambda\bar\lambda\rangle_{\rm oct}|^2\Big),$$

$$\mathcal{D}_3 = \frac{2}{9}\cdot\frac{1}{4}\sum_{\lambda,\bar\lambda}\Big(|\langle ++;\lambda\bar\lambda\rangle_{\rm oct}|^2 + |\langle --;\lambda\bar\lambda\rangle_{\rm oct}|^2 - |\langle +-;\lambda\bar\lambda\rangle_{\rm oct}|^2 - |\langle -+;\lambda\bar\lambda\rangle_{\rm oct}|^2\Big),$$

Mearsurment of Spin-correlation

- **Spin-correlations can be measured through**

$$\frac{1}{\sigma}\frac{\sigma}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4}\left(1 + B_1 A_1 \cos\theta_1 + B_2 A_2 \cos\theta_2 + C A_1 A_2 \cos\theta_1\theta_2\right),$$

  where A_i **is so-called 'spin-analysing power',** ± 1 **for leptons (d-type q),** ∓ 0.41 **for bottom quarks and** ∓ 0.31 **for neutrinos (u-type q).**

- **So we can see that**

$$\begin{aligned} B_1 &\sim \mathcal{D}_1 + \mathcal{D}_2, \\ B_2 &\sim \mathcal{D}_1 - \mathcal{D}_2, \\ C &\sim \mathcal{D}_3. \end{aligned}$$

Parity conserving

- $\mathcal{D}_0$ **leads to the** σ_{tt} **and** A_{FB}**.**

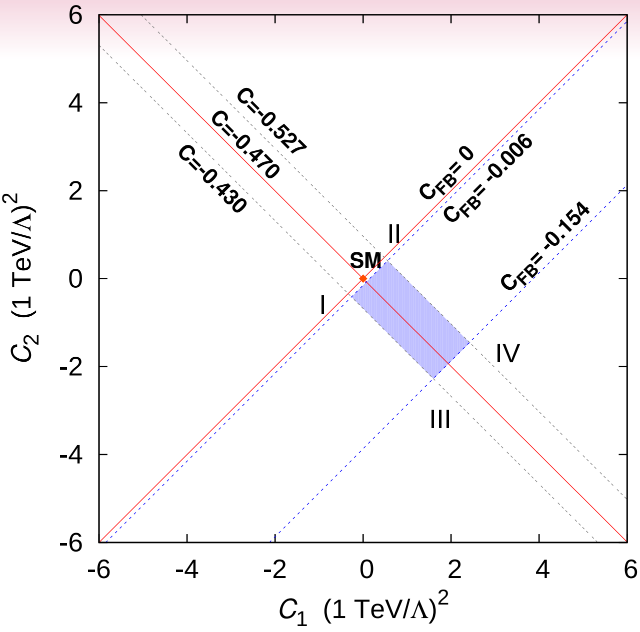
- $\mathcal{D}_3$ **leads to (so-called) spin-correlation variables,**

$$C = \frac{\sigma(t_L\bar{t}_L + t_R\bar{t}_R) - \sigma(t_L\bar{t}_R + t_R\bar{t}_L)}{\sigma(t_L\bar{t}_L + t_R\bar{t}_R) + \sigma(t_L\bar{t}_R + t_R\bar{t}_L)} .$$

- **It depends only on the** $C_1 + C_2$**, so is NOT related to the** $A_{\rm FB}$**.**
- **If we integrate the** $\mathcal{D}_3$ **for the forward/backward directions separately,** $C_{\rm FB}$ **defined as**

$$C_{FB} \equiv C(\cos\theta \geq 0) - C(\cos\theta \leq 0) \qquad (\text{ Just numerator}) ,$$

- **CDF** $C = -0.60 \pm 0.50(stat) \pm 0.16(syst)$ $[0.40]$**, helicity**
  D0 $C = 0.10 \pm 0.45$ $[0.777 + 0.027 - 0.042]$**, beamline**


C=-0.527
C=-0.470
C=-0.430
C_FB= 0
C_FB= -0.006
C_FB= -0.154
SM
I
II
III
IV
6
4
2
0
-2
-4
-6
C_2 (1 TeV/Λ)^2
C_1 (1 TeV/Λ)^2

Parity Violating

- **$\mathcal{D}_1$ is the coefficient of $P_L + \bar{P}_L$, so it is also CP odd.**
- **No such terms in EFT analysis, since the heavy particles are integrated out.**
- **It can arise from the interference between the CP-violating complex couplings together with their decay width etc., if explicit NP form is considered.**

Parity violating, continued

- $\mathcal{D}_2$ **is the coefficient of** $P_L - \bar{P}_L$**, parity odd but CP-even.**

- **Explicitly,**

$$\mathcal{D}_2 \simeq \frac{\hat{s}}{9\,\Lambda^2}\left[(C_1' + C_2')\hat{\beta}_t(1+c_{\hat{\theta}}^2) + (C_1' - C_2')(5-3\hat{\beta}_t^2)c_{\hat{\theta}}\right]$$

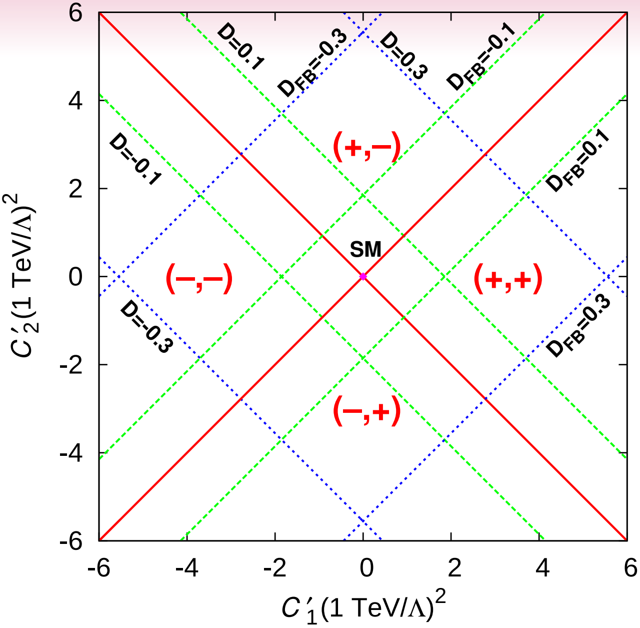
with

$$C_1' \equiv C_{8q}^{RR} - C_{8q}^{LL}\,, \qquad C_2' \equiv C_{8q}^{LR} - C_{8q}^{RL}\,.$$

- **As** σ_{tt}, A_{FB} **and** C, C_{FB}**, new two polarization observables can be drawn.**

$$D \equiv \frac{\sigma(t_R\bar{t}_L) - \sigma(t_L\bar{t}_R)}{\sigma(t_R\bar{t}_R) + \sigma(t_L\bar{t}_L) + \sigma(t_L\bar{t}_R) + \sigma(t_R\bar{t}_L)}\,,$$

$$D_{\rm FB} \equiv D(\cos\hat{\theta} \geq 0) - D(\cos\hat{\theta} \leq 0)$$


D=0.1
D=0.3
D=-0.1
D=-0.3
D_{FB}=-0.3
D_{FB}=-0.1
D_{FB}=0.1
D_{FB}=0.3
(+,−)
(−,−)
(+,+)
(−,+)
SM
$C'_2(1\ \mathrm{TeV}/\Lambda)^2$
$C'_1(1\ \mathrm{TeV}/\Lambda)^2$
6
4
2
0
-2
-4
-6

Model independent vs. Models

- **We can draw some information on each models from model-independent study.**
- **Integrating out heavy fields, the Wilson coefficients are,**

$$\frac{C_{8q}^{RR}}{\Lambda^2} = -\frac{g_{8q}^R g_{8t}^R}{m_{V_{8R}}^2} - \frac{2|\tilde{g}_{1q}^R|^2}{m_{\tilde{V}_{1R}}^2} + \frac{1}{N_C}\frac{|\tilde{g}_{8q}^R|^2}{m_{\tilde{V}_{8R}}^2},$$

$$\frac{C_{8q}^{LL}}{\Lambda^2} = -\frac{g_{8q}^L g_{8t}^L}{m_{V_{8L}}^2} - \frac{2|\tilde{g}_{1q}^L|^2}{m_{\tilde{V}_{1L}}^2} + \frac{1}{N_C}\frac{|\tilde{g}_{8q}^L|^2}{m_{\tilde{V}_{8L}}^2},$$

$$\frac{C_{8q}^{LR}}{\Lambda^2} = -\frac{g_{8q}^L g_{8t}^R}{m_{V_8}^2} - \frac{|\tilde{\eta}_{1q}^L|^2}{m_{\tilde{S}_{1L}}^2} + \frac{1}{2N_c}\frac{|\tilde{\eta}_{8q}^L|^2}{m_{\tilde{S}_{8L}}^2},$$

$$\frac{C_{8q}^{RL}}{\Lambda^2} = -\frac{g_{8q}^R g_{8t}^L}{m_{V_8}^2} - \frac{|\tilde{\eta}_{1q}^R|^2}{m_{\tilde{S}_{1R}}^2} + \frac{1}{2N_c}\frac{|\tilde{\eta}_{8q}^R|^2}{m_{\tilde{S}_{8R}}^2},$$

- **Another interesting possibility is**
  - **color-triplet S_k^γ with mass m_{S_3} and**
  - **color-sextet $S_{ij}^{\alpha\beta}$ with mass m_{S_6} with the SM quarks .**

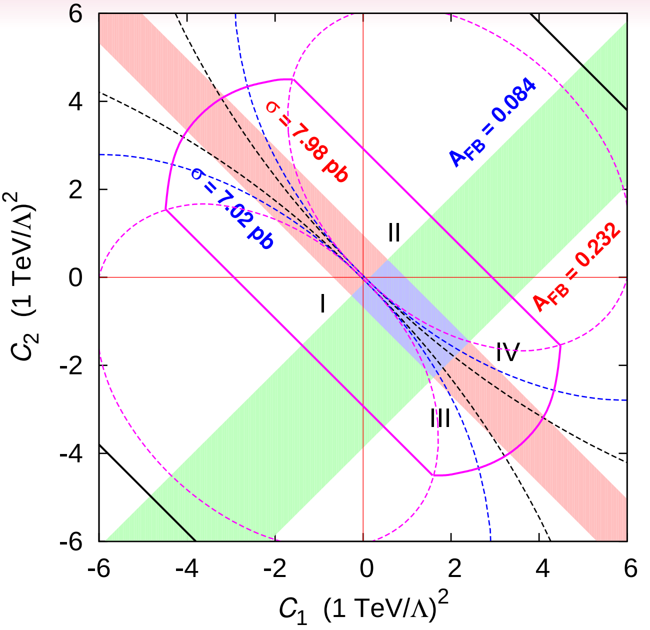
- **With the following interactions**

$$\mathcal{L} = g_s \left[\frac{\eta_3}{2} \epsilon_{\alpha\beta\gamma} \epsilon^{ijk} u_{iR}^\alpha u_{jR}^\beta S_k^\gamma + \eta_6 u_{iR}^\alpha u_{jR}^\beta S_{ij}^{\alpha\beta} + h.c. \right]$$

the $u-$channel exchange of new scalars can contribute to $u\bar{u} \to t\bar{t}$, resulting in

$$\frac{C_{8u}^{RR}}{\Lambda^2} = -\frac{|\eta_3|^2}{m_{S_3}^2} + \frac{2|\eta_6|^2}{m_{S_6}^2}.$$

Resonance	C^{RR}	C^{LL}	C^{LR}	C^{RL}	$C_1 - C_2$	$C_1' + C_2'$	$C_1' - C_2'$	$A_{\rm FB}$
$\tilde{V}_{1R}$	$-$	**0**	**0**	**0**	$-$	$-$	$-$	$\times$
$\tilde{V}_{1L}$	**0**	$-$	**0**	**0**	$-$	$+$	$+$	$\times$
$\tilde{V}_{8R}$	$+$	**0**	**0**	**0**	$+$	$+$	$+$	$\surd$
$\tilde{V}_{8L}$	**0**	$+$	**0**	**0**	$+$	$-$	$-$	$\surd$
$\tilde{S}_{1R}$	0	0	0	$-$	$+$	$+$	$-$	$\surd$
$\tilde{S}_{1L}$	0	0	$-$	0	$+$	$-$	$+$	$\surd$
$\tilde{S}_{8R}$	0	0	0	$+$	$-$	$-$	$+$	$\times$
$\tilde{S}_{8L}$	0	0	$+$	0	$-$	$+$	$-$	$\times$
S_2^{α}	$-$	0	0	0	$-$	$-$	$-$	$\times$
$S_{13}^{\alpha\beta}$	$+$	0	0	0	$+$	$+$	$+$	$\surd$
V_{8R}	$\pm$	0	0	0	$\pm$	$\pm$	$\pm$	$\surd(+)$ **or** $\times(-)$
V_{8L}	0	$\pm$	0	0	$\pm$	$\mp$	$\mp$	$\surd(+)$ **or** $\times(-)$
V_{8R} , V_{8L}	**indef.**	**indef.**	**indef.**	**indef.**	**indef.**	**indef.**	**indef.**	**indef.**



6
4
2
0
-2
-4
-6
-6 -4 -2 0 2 4 6
C_1 (1 TeV/Λ)^2
C_2 (1 TeV/Λ)^2
σ = 7.98 pb
σ = 7.02 pb
A_{FB} = 0.084
A_{FB} = 0.232
I
II
III
IV

Simple cases

- **Following four cases are simple :**

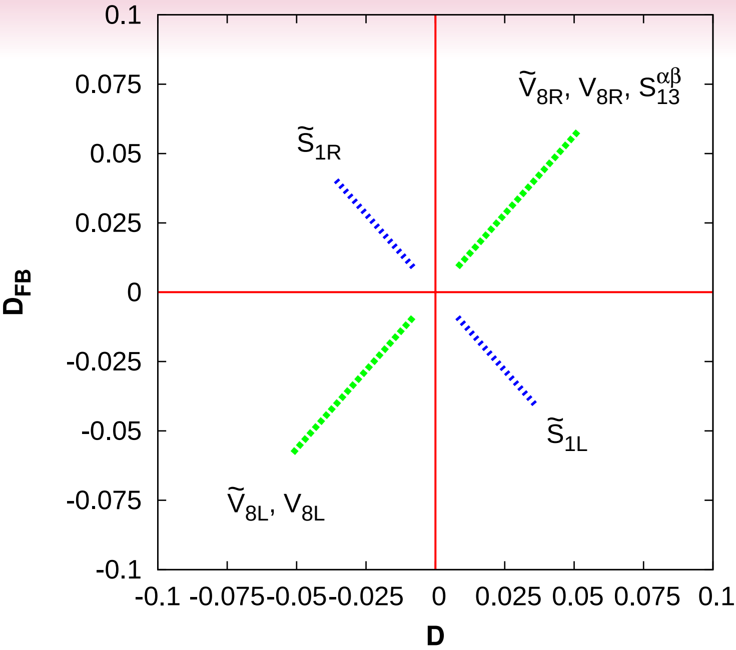
$$V_{8R,8L} \quad : \quad -\frac{g_{8q}^{R,L} g_{8t}^{R,L}}{m_{V_{8R,L}}^2} \simeq 0.649 \pm 0.024, \ ,$$

$$\tilde{V}_{8R,8L} \quad : \quad \frac{1}{N_c}\left(\frac{1\,\mathrm{TeV}}{m_{\tilde{V}_{8R,8L}}}\right)^2 |\tilde{g}_{8q}^{R,L}|^2 \simeq 0.649 \pm 0.024\,,$$

$$\tilde{S}_{1R,1L} \quad : \quad \left(\frac{1\,\mathrm{TeV}}{m_{\tilde{S}_{1R,1L}}}\right)^2 |\tilde{\eta}_{1q}^{R,L}|^2 \simeq 0.799 \pm 0.174\,,$$

$$S_{13}^{\alpha\beta} \quad : \quad 2\left(\frac{1\,\mathrm{TeV}}{m_{S_6}}\right)^2 |\eta_6|^2 \simeq 0.649 \pm 0.024 \ .$$

- **New spin-correlation observables D and D_{FB} values are predicted for each case.**


$\tilde{V}_{8R}, V_{8R}, S_{13}^{\alpha\beta}$
$\tilde{S}_{1R}$
$\tilde{S}_{1L}$
$\tilde{V}_{8L}, V_{8L}$
0.1
0.075
0.05
0.025
0
-0.025
-0.05
-0.075
-0.1
-0.1 -0.075 -0.05 -0.025 0 0.025 0.05 0.075 0.1
D_{FB}
D

More general case

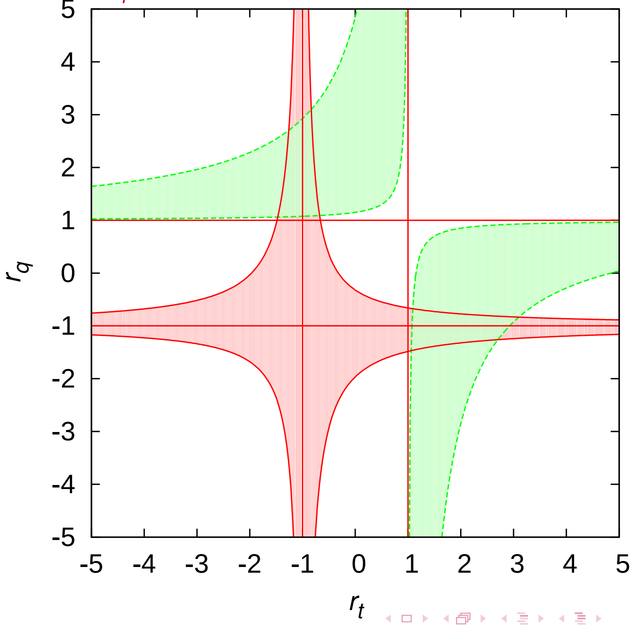
- **In general, V_{8L} and V_{8R} can coexist. In that case the new observables are not so definite compared to the previous cases.**

- **We introduce new parameterizations**

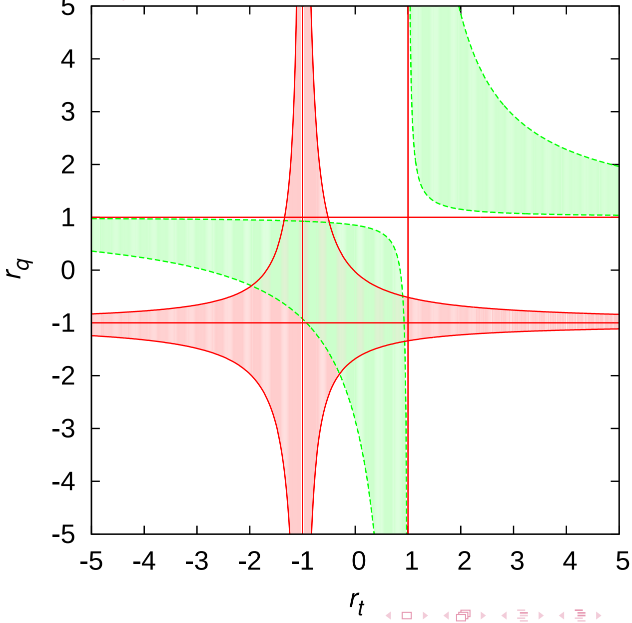
$$
\begin{aligned}
(C_1 + C_2)/\Lambda^2 &= -g^L_{8q} g^L_{8t} (r_q + 1)(r_t + 1)/m^2_{V8} \\
(C_1 - C_2)/\Lambda^2 &= -g^L_{8q} g^L_{8t} (r_q - 1)(r_t - 1)/m^2_{V8} \\
(C'_1 + C'_2)/\Lambda^2 &= -g^L_{8q} g^L_{8t} (r_q + 1)(r_t - 1)/m^2_{V8} \\
(C'_1 - C'_2)/\Lambda^2 &= -g^L_{8q} g^L_{8t} (r_q - 1)(r_t + 1)/m^2_{V8}
\end{aligned}
$$

with $r_q \equiv g^R_{8q}/g^L_{8q}$ and $r_t \equiv g^R_{8t}/g^L_{8t}$. Any deviation of r_q (r_t) from 1 characterizes P violation in the light (top) quark sector.

$g^L_{8q} g^L_{8t} \, (1\,\mathrm{TeV}/m_{V8})^2 = +1,$



$g^L_{8q} g^L_{8t} \, (1\,\mathrm{TeV}/m_{V8})^2 = -1,$



Important observations are

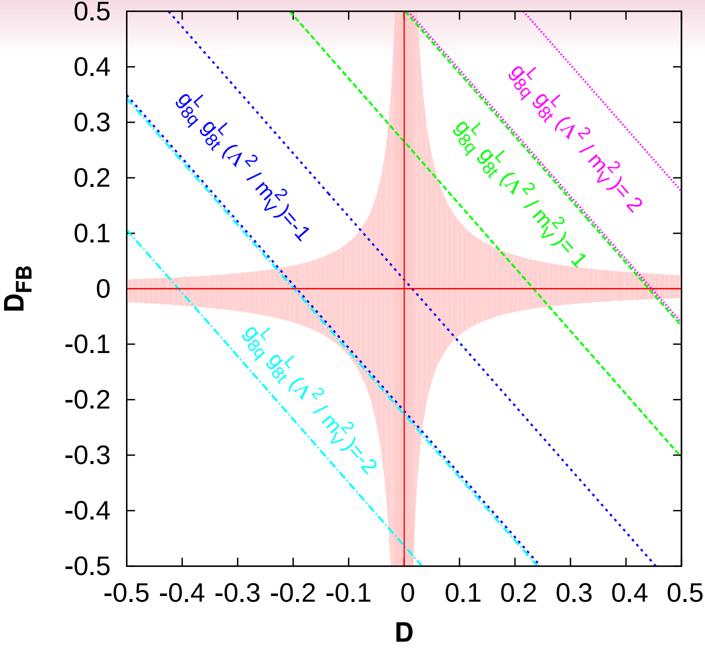
- **By observing the relation** $(C_1 + C_2)(C_1 - C_2) = (C_1' + C_2')(C_1' - C_2')$ **which leads to**

$$\Delta\sigma_{t\bar{t}}\, \Delta A_{\mathrm{FB}} \propto D\, D_{\mathrm{FB}}\,.$$

Let us note that $\Delta\sigma_{t\bar{t}} \propto (C_1 + C_2)$**,** $\Delta A_{\mathrm{FB}} \propto (C_1 + C_2)$**,** $D \propto (C_1' + C_2')$**, and** $D_{\mathrm{FB}} \propto (C_1' - C_2')$**.**

- **Furthermore, we observe**

$$\begin{aligned} & g_{8q}^L g_{8t}^L \left(\frac{\Lambda}{m_{V8}}\right)^2 \\ = & \frac{\left[(C_1' + C_2') - (C_1 + C_2)\right]\left[(C_1' + C_2') - (C_1 - C_2)\right]}{4(C_1' + C_2')} \\ = & \frac{1}{4}\left[(C_1' + C_2') - (C_1 + C_2) - (C_1 - C_2) + (C_1' - C_2')\right]\,, \end{aligned}$$



$g^L_{8q}\, g^L_{8t}\, (\Lambda^2 / m_V^2) = -1$
$g^L_{8q}\, g^L_{8t}\, (\Lambda^2 / m_V^2) = 1$
$g^L_{8q}\, g^L_{8t}\, (\Lambda^2 / m_V^2) = 2$
$g^L_{8q}\, g^L_{8t}\, (\Lambda^2 / m_V^2) = -2$
D_{FB}
D
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5

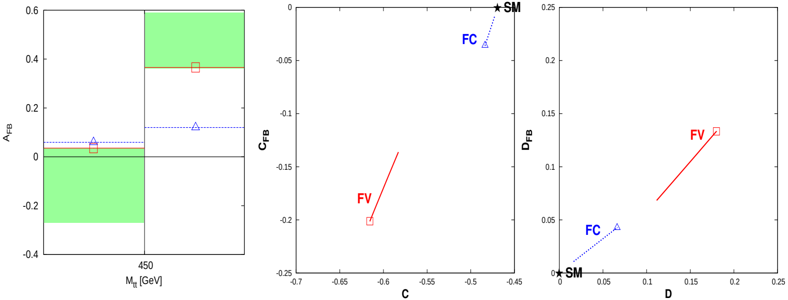
- **In case of rather light NP particles, we should perform the analysis with the FULL theory. (in progress)**
- **As an example, motivated by Wjj anomaly, we consider flavor conserving/violating vector particle with mass 150 GeV.**
- **The relevant Lagrangian is**

$$\mathcal{L}_{\rm NP} = - g_s \sum_{q=u,d,t} \overline{q}\, \gamma^\mu \left(g_L^q P_L + g_R^q P_R \right) q\, V_\mu - \left[g_s \overline{u} \gamma^\mu \left(\tilde{g}_L^t P_L + \tilde{g}_R^t P_R \right) t\, V_\mu + h.c. \right] .$$

- **The allowed range for the couplings are**

$$0.67 \lesssim g_R^t \lesssim 1.4,$$
$$0.48 \lesssim \tilde{g}_R^t \lesssim 0.51.$$

- **Other couplings are rather fixed by Wjj and UA2 constraints.**



0.6
0.4
0.2
0
-0.2
-0.4
A_FB
450
M_tt [GeV]
SM
FC
FV
C_FB
C
-0.7 -0.65 -0.6 -0.55 -0.5 -0.45
0 -0.05 -0.1 -0.15 -0.2 -0.25
FV
FC
SM
D_FB
D
0 0.05 0.1 0.15 0.2 0.25

Summary and Prospect

- **We performed the model independent analysis of $t\bar{t}$ productions at Tevatron using D6 contact interactions relevant to $q\bar{q} \to t\bar{t}$.**

- **We showed the allowed region which fit the cross section and $A_{\rm FB}$ altogether.**

- **More observables are proposed and predicted, which are crucial for revealing the chiral structure of the relevant new physics.**

- **Implimenation of QCD NLO results must be done for spin-polarization variables.**

- **Complete and general analysis with FULL theory is in progress, including 16 observables ($\mathcal{D}_1$ and more) in relation with the LHC signals.**

謝謝敬聽!