Conditions for reciprocal cooperation with finite memories

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Mechanisms for cooperation

- Kin selection

  "I would save two brothers or eight cousins."

  - J.B.S. Haldane

- Direct reciprocity

  "I scratch your back
  and you'll scratch mine."

- Just an old story?

Main points of this work

1. Generic payoff structure ($0<c<1$)

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ 1+c & c \end{pmatrix} \end{array}$$

2. Effects of memory
   I. Deterministic strategies
   II. Stochastic strategies

I. Deterministic strategies

- M_0 : never mind the history

  ex) Always defect

- $M_{1/2}$: reactive to the other's last move

  ex) Do the same. (Tit-for-tat)

- M_1 : memory of length one

  ex) If it worked, do it again. (Win-stay-lose-shift)

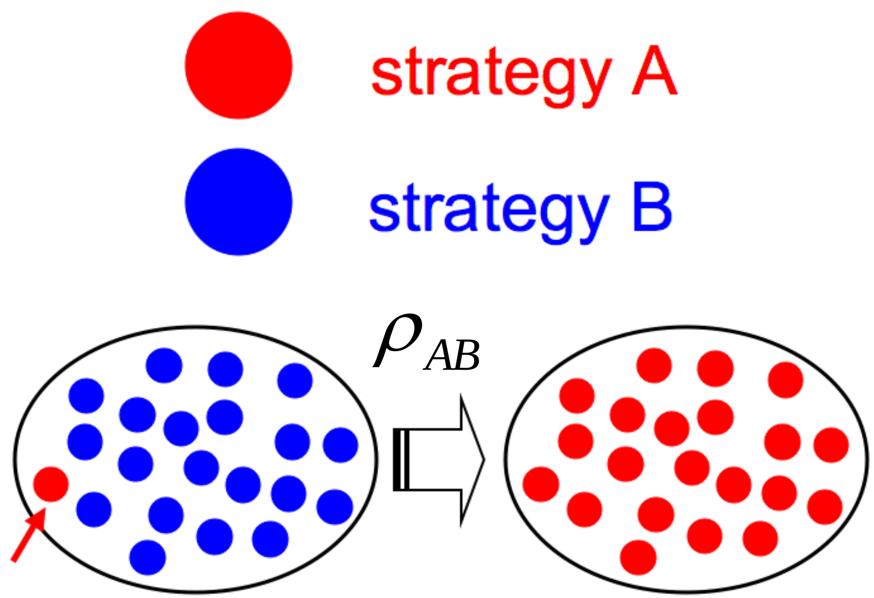
  Win-stay

  Cooperate if (C,C) or (D,D)

  Lose-shift

Moran process

- Finite population (say, N=100)
- Fitness = 1 + w*payoff (w: selection strength)
- Fixation

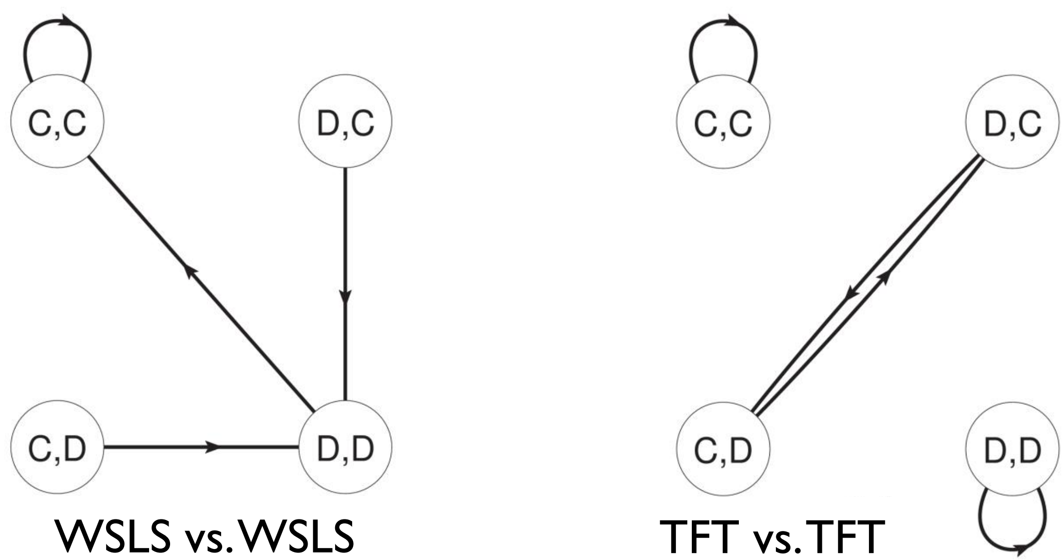


Six time scales

- 1 : Each round of the game
  τ_e : Error ~ $1/e$
  τ_u : Strategy update
  τ_f : Fixation
  τ_μ : Mutation ~ $1/\mu$
  τ_{obs} : Observation
- Assume
  $1 \ll \tau_e \ll \tau_u < \tau_f \ll \tau_\mu \ll \tau_{obs}$

Error vs. Update ($\tau_e \ll \tau_u$)

- Strategy updates based on error-averaged payoffs.

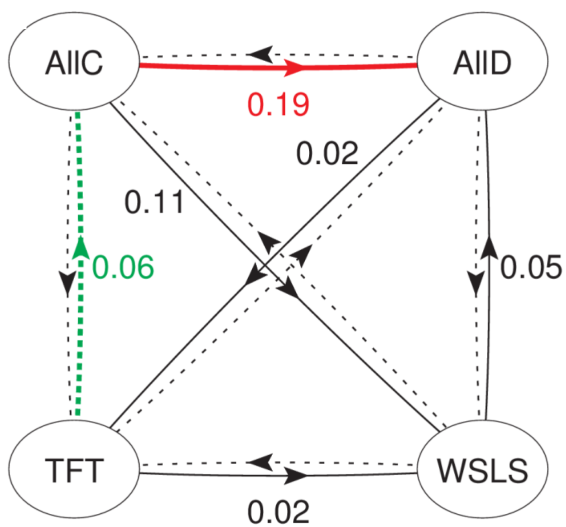


- Unique eigenvector for every i and j:

$$\mathbf{h}^{ij} = (h^{ij}_{CC}, h^{ij}_{CD}, h^{ij}_{DC}, h^{ij}_{DD})$$

Fixation vs. Mutation ($\tau_f \ll \tau_\mu$)

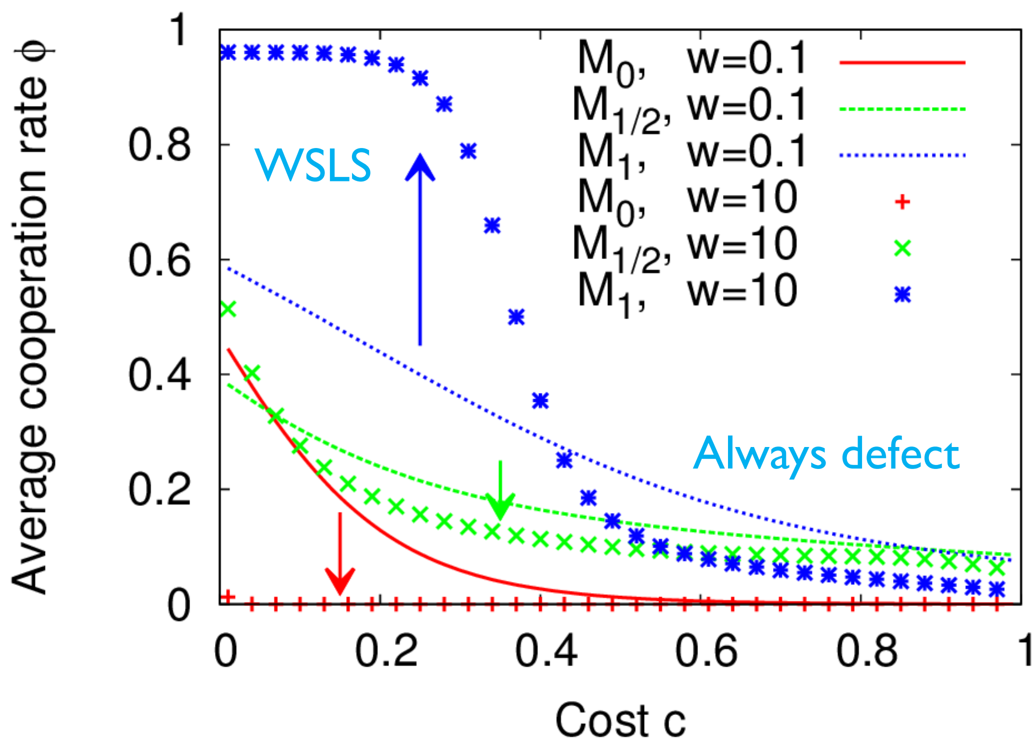
- Each fixation event is separated.
- From strategy B to A with probability ρ_{AB}



- Observe fixations $\tau_{obs} \gg \tau_\mu \rightarrow$ Eigenvector

Memory for cooperation = 1

- Average cooperation rate



Memory-2 deterministic strategies

- Error can be handled explicitly.
- Top-down selection with three conditions
  - Efficiency: self-cooperating
  - Defensibility: against any strategy
  - Distinguishability: from naïve cooperators
- Behavioral rule
  1. Play TFT.
  2. Play Anti-TFT if you made a mistake, until
     1. cooperation is recovered.
     2. the opponent is ungrateful to your cooperation.

II. Stochastic strategies

- P_{XY}: Probability to cooperate when (I, You) = (X, Y)
- Definition of a strategy

$$\mathbf{i} = (P_{CC}, P_{CD}, P_{DC}, P_{DD})$$

- $p(\mathbf{i}, \mathbf{j}; e)$:Error-averaged payoff of **i** against **j**
- Uncountably many!

Determinant [T. Antal et al. (2014)]

- Abundance under weak selection

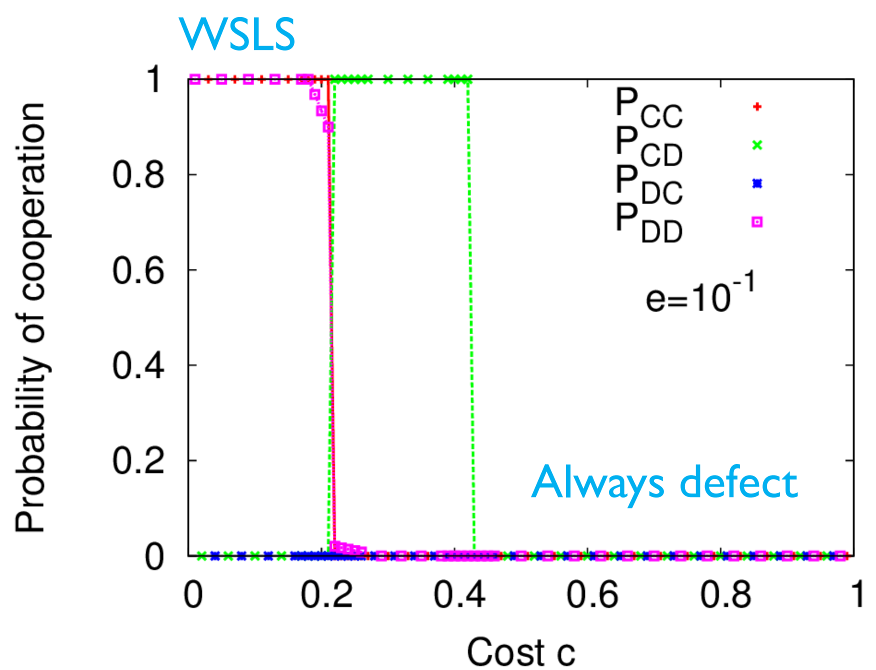
$$L(\mathbf{i}, e) = \int d\mathbf{j} [p(\mathbf{i}, \mathbf{i}; e) + p(\mathbf{i}, \mathbf{j}; e) - p(\mathbf{j}, \mathbf{i}; e) - p(\mathbf{j}, \mathbf{j}; e)]$$

- Find the most abundant strategy:

$$\mathbf{i}^* = \arg\min_{\mathbf{i}} L(\mathbf{i}, e)$$

Most abundant strategies

- High cost : (0,0,0,0) = Always defect
  Low cost : (1,0,0,1) ≈ WSLS
  Medium cost : (0,1,0,0) = ???



(0,1,0,0)?

- Cooperate if betrayed

$$(I, You) = (C,C) \to D$$
$$(C,D) \to C$$
$$(D,C) \to D$$
$$(D,D) \to D$$

- Self-interaction $\mathbf{h}^{ii} = \left(0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}\right)$

- Uncomfortable?

  cf. Stewart and Plotkin (2013)

Comparison with Always Defect

Strategy $\mathbf{i} = (P_{CC}, P_{CD}, P_{DC}, P_{DD})$	**Always defect** **(0,0,0,0)**	**Cooperate if betrayed** **(0,1,0,0)**
Self-interaction	$\mathbf{h}^{ii} = (0,0,0,1)$	$\mathbf{h}^{ii} = \left(0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}\right)$
Against $\mathbf{j} = (P_{CC}, P_{CD}, P_{DC}, P_{DD})$	$\mathbf{h}^{ij} = \left(0,0, \frac{P_{DD}}{1+P_{DD}-P_{CD}}, \frac{1-P_{CD}}{1+P_{DD}-P_{CD}}\right)$	

Self vs. Non-self

Conclusion

- Abundance via time-scale separation
- Full memory of the last time step
- Low cost: Win-Stay-Lose-Shift
  (Direct reciprocity)
  Medium cost: Cooperate if betrayed
  (Kin selection)
  High cost: Always defect

Collaborators

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