Extracting work from random collisions: A model of a quantum heat engine

Vahid Shaghaghi

Center for Theoretical Physics of Complex Systems(PCS), Institute for Basic Physics(IBS), South Korea

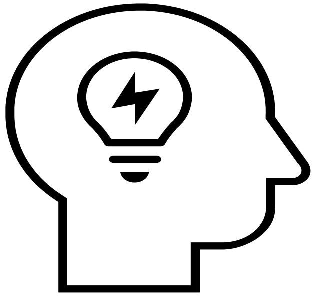
Center for Nonlinear and Complex Systems, University of Insubria, Italy

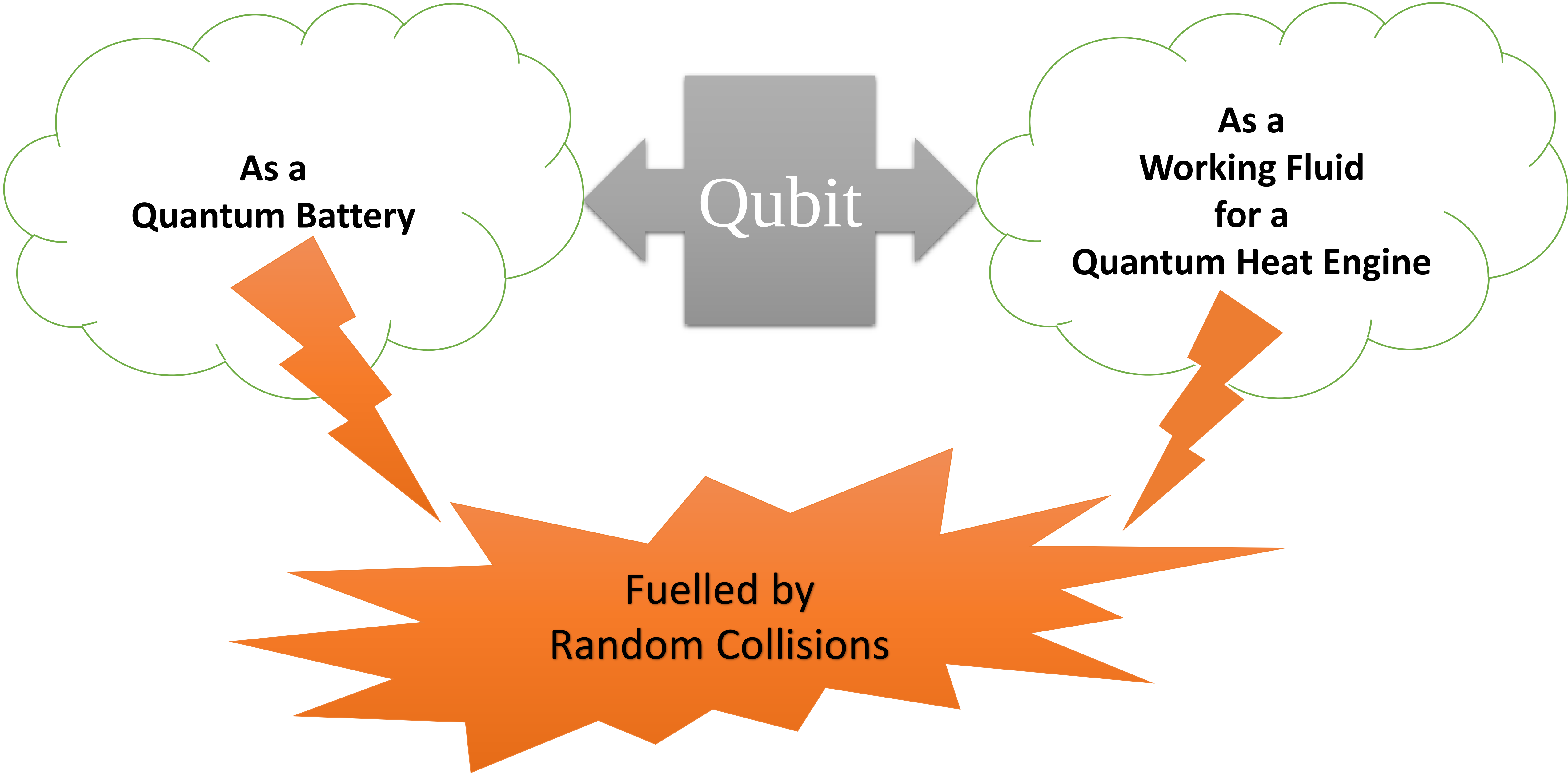
Collision Models can describe unitary interactions even in the strong coupling regime

Random Collisions are a special type of collision models

Thanks to the random nature of the collisions, they are cheap resource

Can random collisions be exploited as a useful resource to perform quantum thermodynamics tasks?

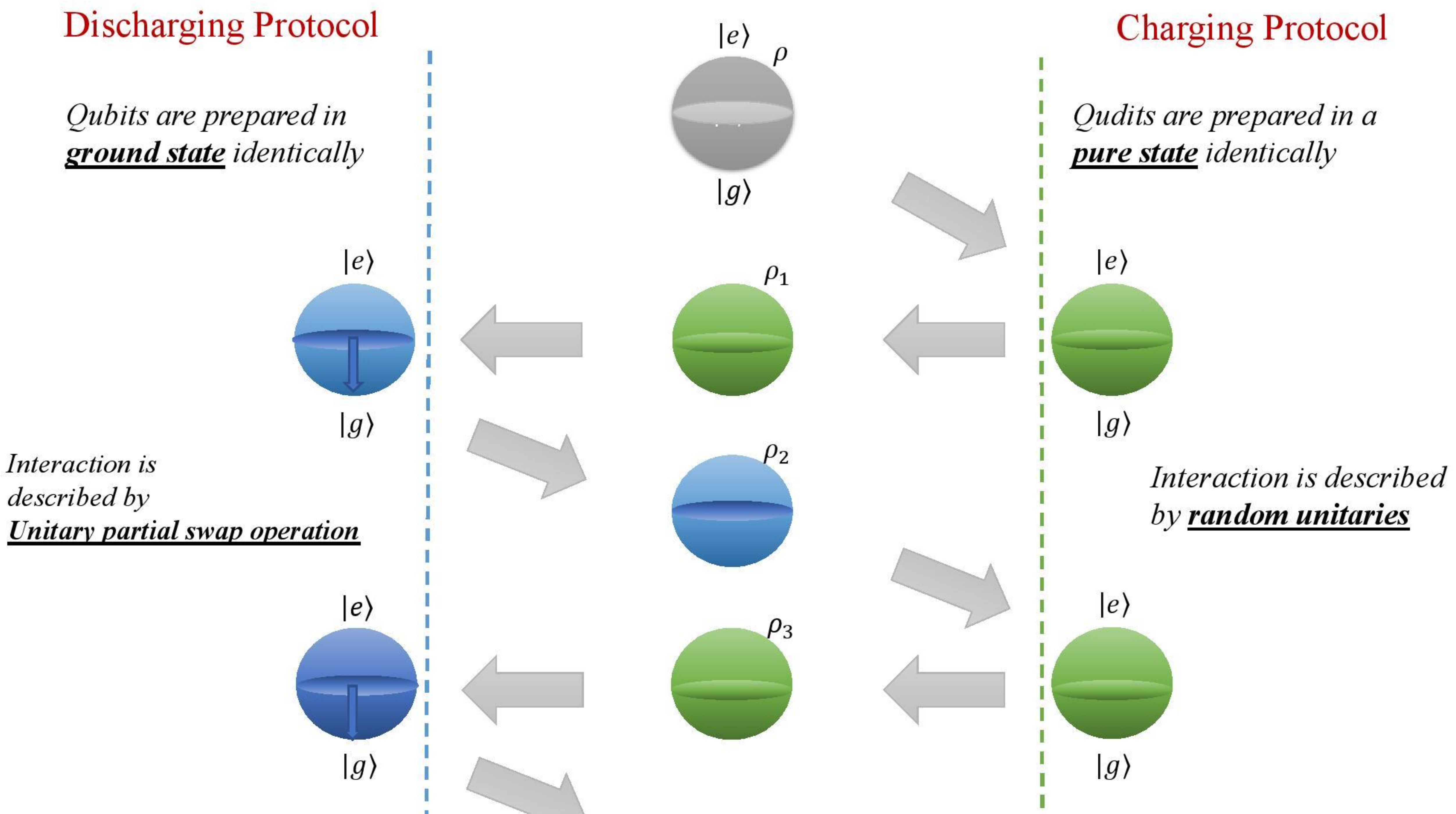



Qubit
As a
Quantum Battery
As a
Working Fluid
for a
Quantum Heat Engine
Fuelled by
Random Collisions

First Part:

Statistical distribution of the ergotropy

A Single Qubit
as a
Quantum Battery


Discharging Protocol
Charging Protocol
Qubits are prepared in **ground state** identically
Qudits are prepared in a **pure state** identically
Interaction is described by **Unitary partial swap operation**
Interaction is described by **random unitaries**
$|e\rangle$
$|g\rangle$
ρ
ρ_1
ρ_2
ρ_3

Formalism

- **Charging Protocol**

➢ *Interaction is described by* ***Random Unitaries***

➢ *Unitary Group U(2L) is parametrized by the* ***Hurwitz representation***

➢ *Collisions are not weak, i.e. they can* ***strongly*** *change the energy and the coherences of the qubit battery*

- **Discharging Protocol**

➢ *Interaction is described by* ***unitary partial swap operation***

$$\hat{P}(\alpha) = \cos\alpha + i\sin\alpha\,\hat{S} \qquad 0 \leq \alpha \leq \frac{\pi}{2}$$

$\hat{S}$ *is the swap operator:* $\hat{S}\{|\emptyset\rangle \otimes |\Psi\rangle\} = |\Psi\rangle \otimes |\emptyset\rangle$

- **Maximum Extractable Work from the Qubit Battery**

➢ *Qubit Hamiltonian :* $\hat{H} = \frac{1}{2}\Delta\sigma_z$

➢ *Qubit state in the Bloch sphere representation:* $\rho = \frac{1}{2}(I + r.\sigma)$

➢ ***Ergotropy:*** $E(\rho) = \Delta(r + z)$

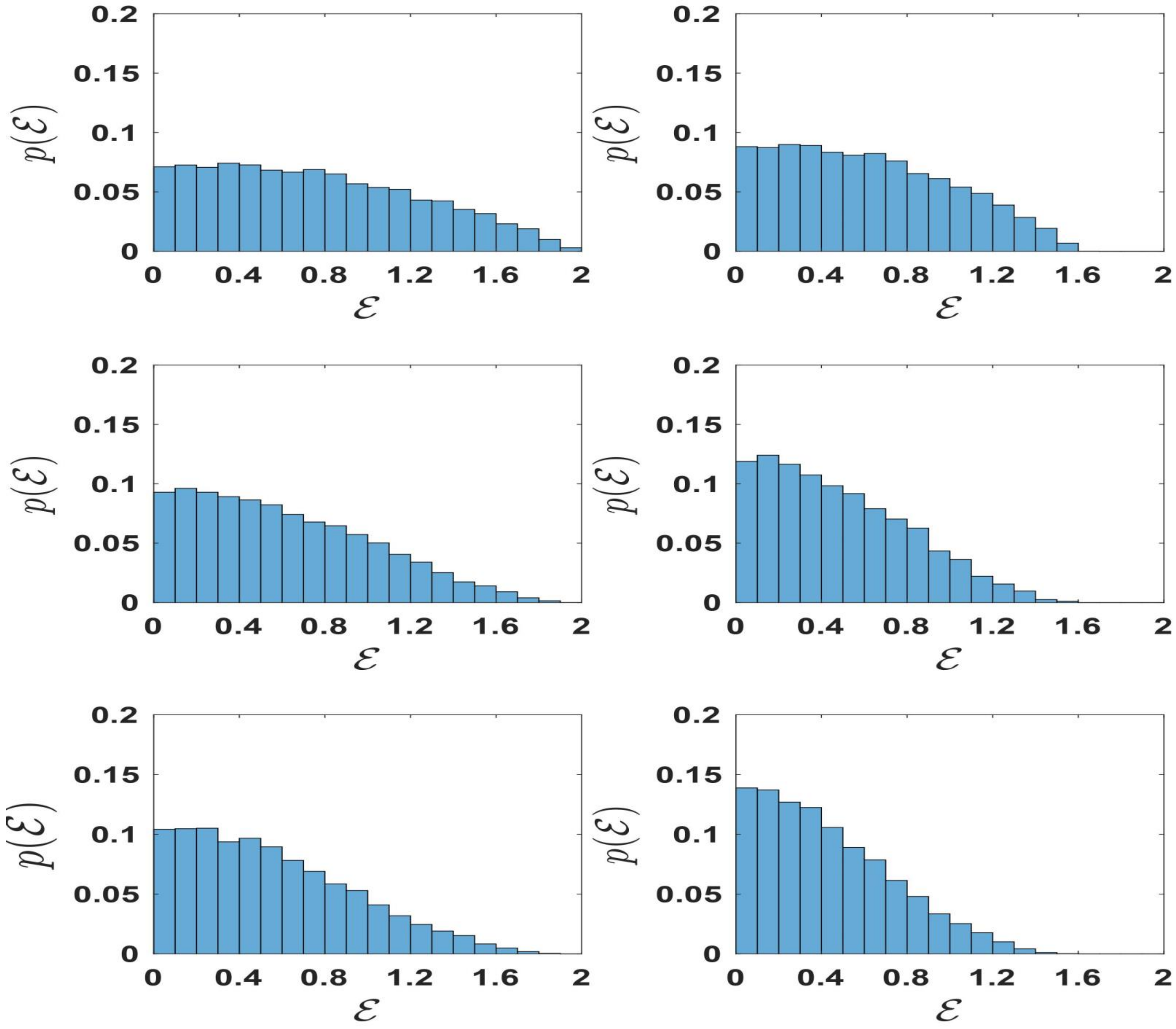
Results

statistical distribution of the ergotropies

Left Panels:
after the first, the third, and the fifth collision
collisions with the hot reservoir
L=2

Right Panels:
after the second, the fourth, and the sixth collision
collisions with the cold reservoir

$$\alpha = \frac{\pi}{10}$$



Results

ensemble-averaged ergotropy as a function of the number of collisions

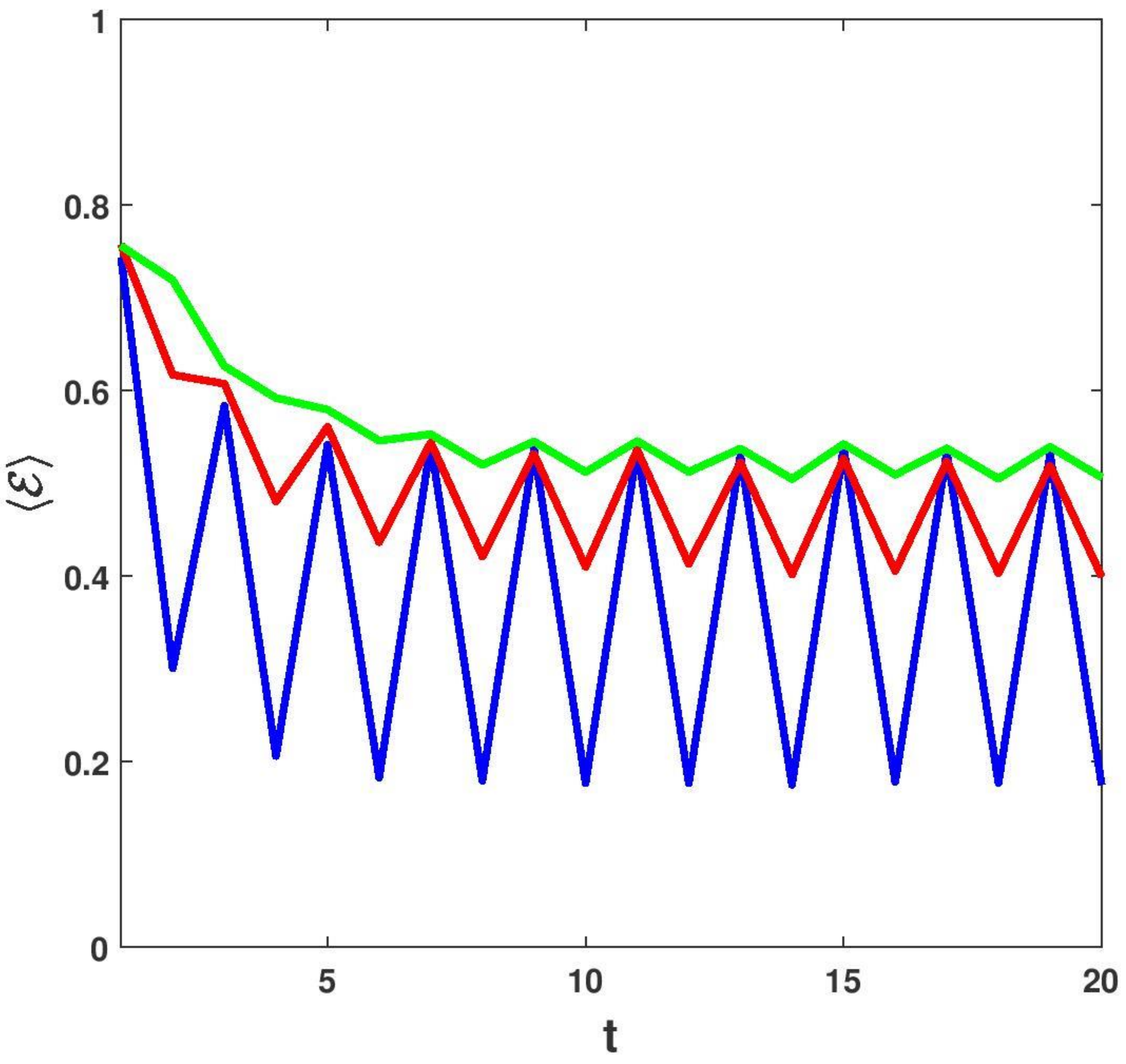
dimension of the hot reservoir is fixed L=2

$\alpha = \frac{\pi}{20}$(green line)

$\alpha = \frac{\pi}{10}$ (red line)

$\alpha = \frac{\pi}{5}$ (blue line)

increasing α=faster achieving steady state and passive state



Results

statistical distribution of the ergotropies after six cycles

swap parameter is fixed

$$\alpha = \frac{\pi}{10}$$

Right Panels:

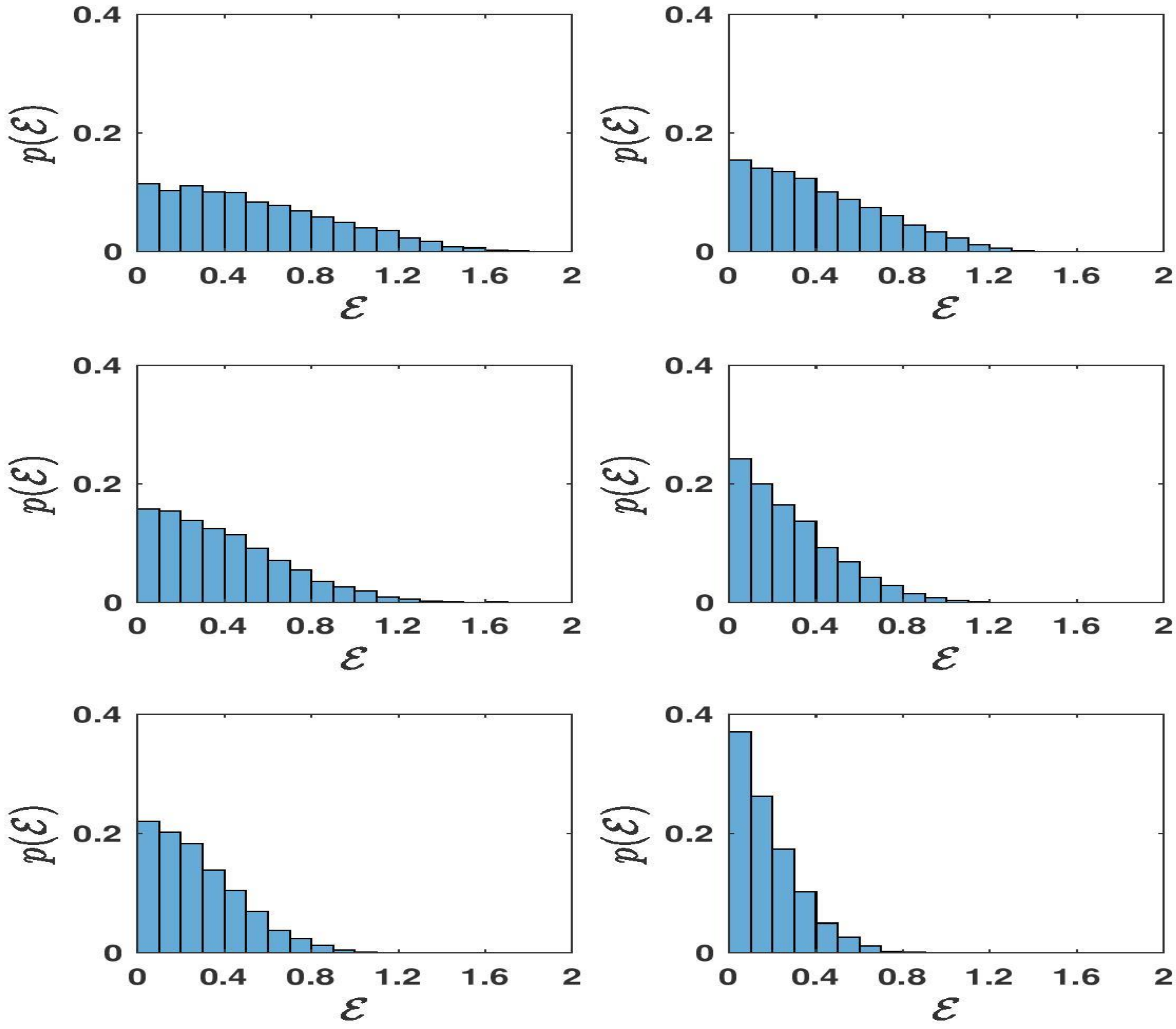
collisions with the cold reservoir

Left Panels:

collisions with the hot reservoir

L=2, 4, 8

distribution shrinks with increasing the hot reservoir dimension



Results

ensemble-averaged ergotropy as a function of the number of collisions

swap parameter is fixed

$$\alpha = \frac{\pi}{10}$$

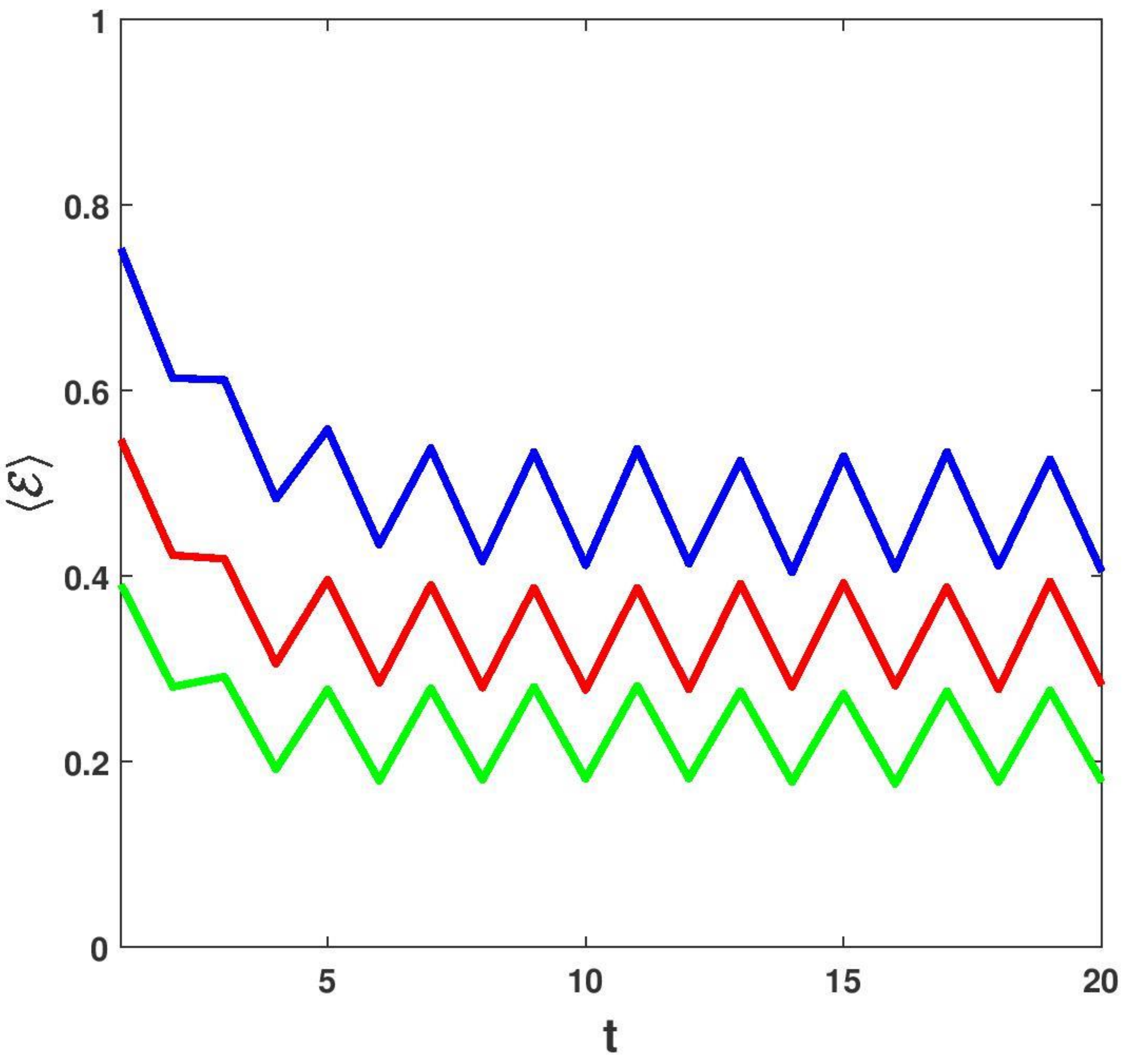
L=2 (blue line)

L=4 (red line)

L=8 (green line)

$$L \to \infty = T \to \infty$$

$$\rho \to \frac{1}{2}[|g\rangle\langle g| + |e\rangle\langle e|]$$



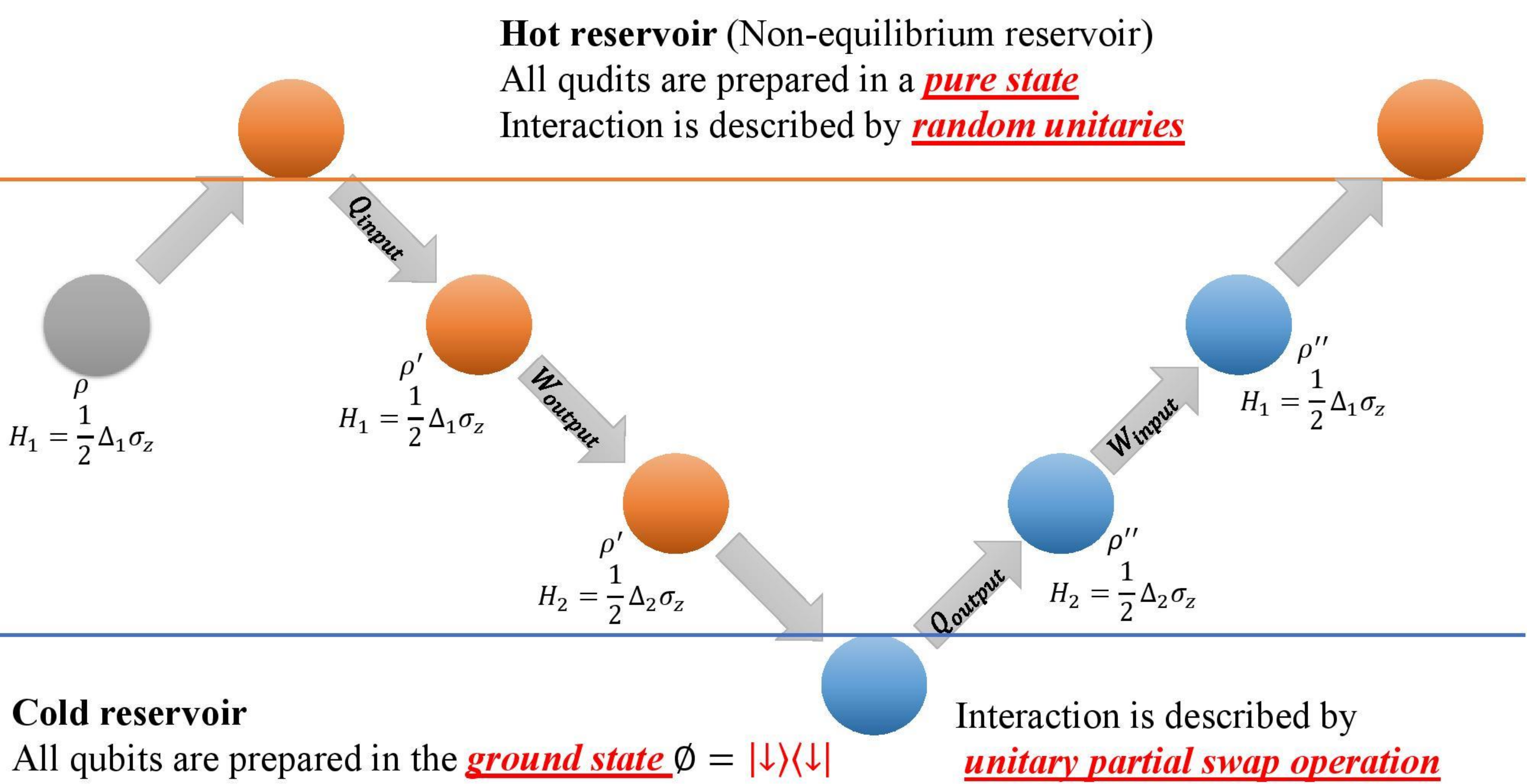
Second Part:

Statistical distribution of the efficiency

A single Qubit as a Working Fluid

For

A Quantum Heat Engine


Hot reservoir (Non-equilibrium reservoir)
All qudits are prepared in a pure state
Interaction is described by random unitaries
Q_{input}
W_{output}
W_{input}
Q_{output}
ρ
$H_1 = \frac{1}{2}\Delta_1\sigma_z$
ρ'
$H_1 = \frac{1}{2}\Delta_1\sigma_z$
ρ'
$H_2 = \frac{1}{2}\Delta_2\sigma_z$
ρ''
$H_2 = \frac{1}{2}\Delta_2\sigma_z$
ρ''
$H_1 = \frac{1}{2}\Delta_1\sigma_z$
Cold reservoir
All qubits are prepared in the ground state $\emptyset = |\downarrow\rangle\langle\downarrow|$
Interaction is described by
unitary partial swap operation

Formalism
Quantum Otto Engine

Stroke A: *the system interacts with the nonequilibrium reservoir*

$$\rho \rightarrow \rho'$$

Absorbed heat $Q_{in} = Tr(H_1\rho') - Tr(H_1\rho) = \frac{1}{2}\Delta_1(z' - z)$

Stroke B: *the system interacts with an external agent*

$$H_1 = \frac{1}{2}\Delta_1 \rightarrow H_2 = \frac{1}{2}\Delta_2 \qquad \Delta_1 > \Delta_2 > 0$$

work performed by the system $W_{out} = Tr(H_1\rho') - Tr(H_2\rho') = \frac{1}{2}z'(\Delta_1 - \Delta_2)$

Stroke C: *the system interacts with the cold reservoir*

$$\rho' \rightarrow \rho''$$

released heat $Q_{out} = Tr(H_2\rho'') - Tr(H_2\rho') = \frac{1}{2}\Delta_2(z'' - z')$

Stroke B: *the system interacts with an external agent*

$$H_2 = \frac{1}{2}\Delta_2 \rightarrow H_1 = \frac{1}{2}\Delta_1 \qquad \Delta_1 > \Delta_2 > 0$$

work performed on the system $W_{in} = Tr(H_2\rho'') - Tr(H_1\rho'') = \frac{1}{2}z''(\Delta_2 - \Delta_1)$

Single realization efficiency

$$\eta = \frac{W_{out} + W_{in}}{Q_{in}} = \frac{z' - z''}{z' - z}\left(1 - \frac{\Delta_2}{\Delta_1}\right)$$

Ensemble-averaged Efficiency(Standard otto engine efficiency)

$$\eta_m = \frac{\langle W_{out} + W_{in}\rangle}{\langle Q_{in}\rangle} = 1 - \frac{\Delta_2}{\Delta_1}$$

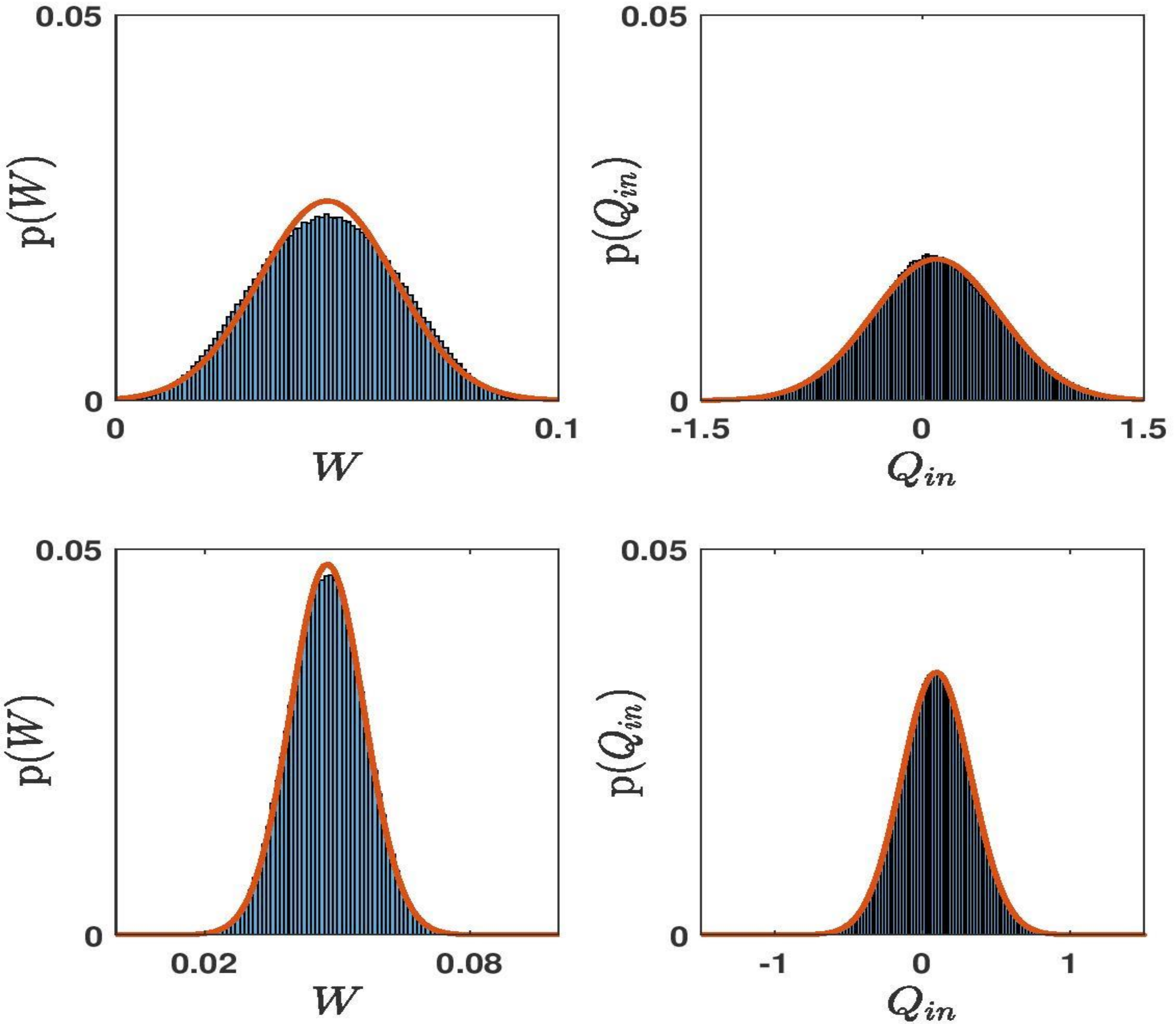
statistical distribution
work and input heat

$$\alpha = \frac{\pi}{10}$$

$$\Delta_1 = 2, \Delta_2 = 1$$

Top panels:
L=2
Bottom panels:
L=8

red lines show Gaussian fits
standard deviation decreases as $\frac{1}{\sqrt{L}}$



Results

Statistical distribution of efficiency

Gaussian distribution → power-law decay

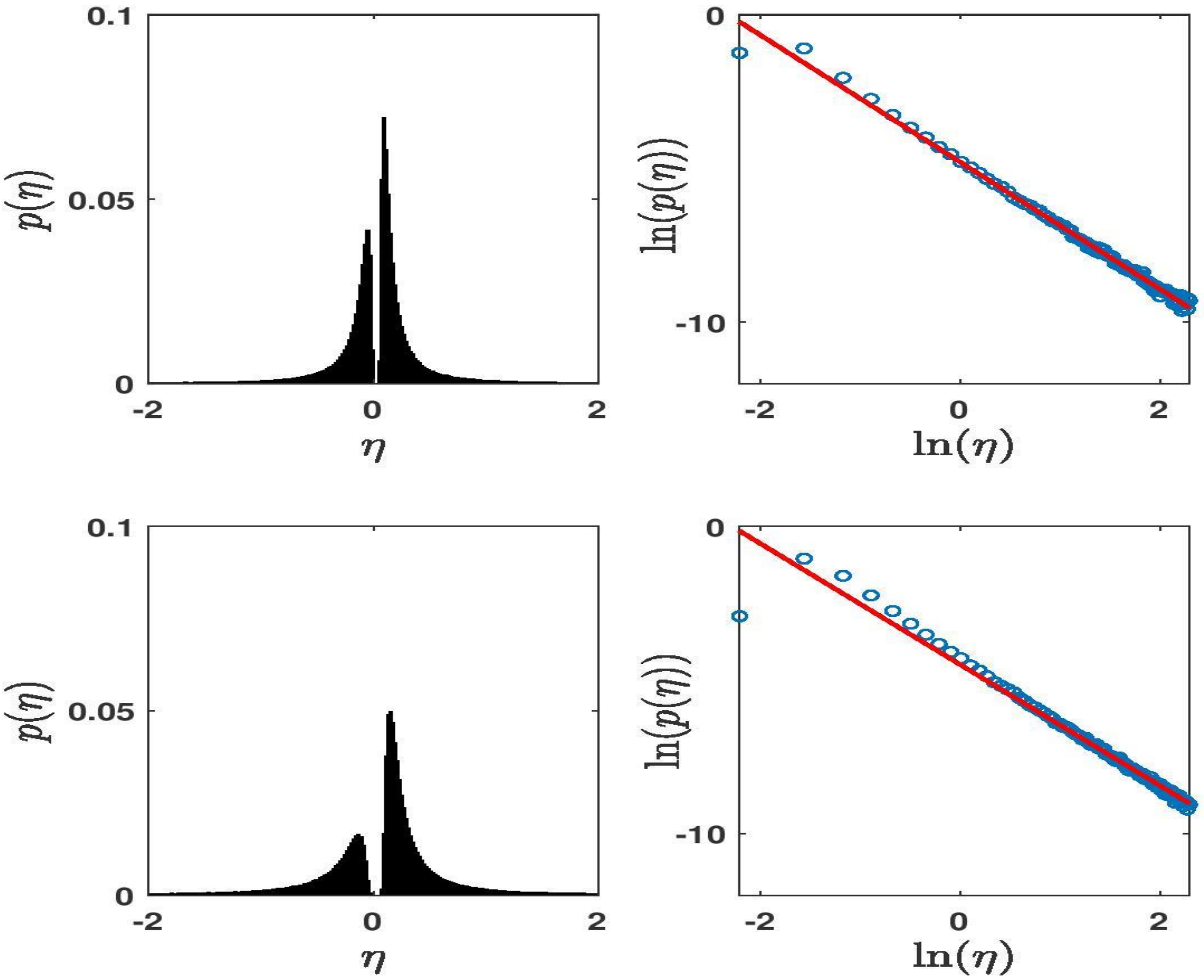
$p(\eta) \propto {}^{1}/_{\eta^2}$

$p(\eta) \propto {}^{1}/_{\eta^{\beta}}$

Top panels:
L=2
$\beta = -2$

Bottom panels:
L=8
$\beta = -1.97$

mean efficiency does not coincide with macroscopic efficiency



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✓ *M. Campisi, P. Hänggi, and P. Talkner, Rev. Mod. Phys. 83, 771 (2011)*

Giuliano Benenti
University of Insubria, Italy

Massimo Palma
University of Palermo, Italy

V. Shaghaghi, G.M. Palma, G. Benenti

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Thanks for your attention