The Neutrino Mass Hierarchy and cos(δ) from Reactor Neutrino Experiments

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RENO 50 - Towards the Mass Hierarchy

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Based on work done in collaboration with:
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θ_{13} and the neutrino mass hierarchy

A nuclear reactor emits $\overline{\nu}_e$ which then oscillate.

Measuring these neutrinos, for example with inverse β decay, one can determine the electron neutrino survival probability as a function of energy E.

This survival probability depends upon the neutrino mass hierarchy and so a reactor neutrino experiment can in principle determine the mass hierarchy (Petcov and Piai, 2002).

The reactor neutrino spectrum at baselines shorter than 25 km is essentially independent of the neutrino mass hierarchy (Petcov and Piai, 2002; Choubey et al, 2003)

The observed reactor neutrino spectrum at a medium baseline manifests 1-2 oscillations on large scales and a fine structure with amplitude $\sin^2(2\theta_{13})$ of 1-3 oscillations, perturbed by 2-3 oscillations.

3 Flavor Oscillations

$$
\begin{aligned}
P_{ee} &= |\langle \nu_e | \exp\left(i \frac{\mathbf{M}^2 L}{2E} \right) | \nu_e \rangle|^2 \\
&= \sin^4(\theta_{13}) + \cos^4(\theta_{12})\cos^4(\theta_{13}) + \sin^4(\theta_{12})\cos^4(\theta_{13}) \\
&\quad + \frac{1}{2}(P_{12} + P_{13} + P_{23}) \\
P_{12} &= \sin^2(2\theta_{12})\cos^4(\theta_{13}) \cos\left(\frac{\Delta M^2_{21} L}{2E} \right) \\
P_{13} &= \cos^2(\theta_{12})\sin^2(2\theta_{13}) \cos\left(\frac{|\Delta M^2_{31}| L}{2E} \right) \\
P_{23} &= \sin^2(\theta_{12})\sin^2(2\theta_{13}) \cos\left(\frac{|\Delta M^2_{32}| L}{2E} \right)
\end{aligned}
$$

So the fine structure consists of 1-3 oscillations P_{13} perturbed by 2-3 oscillations P_{23} which have a slightly different wavenumber, leading to beats at the 1-2 wavenumber.

Reactor electron antineutrino spectrum with oscillations

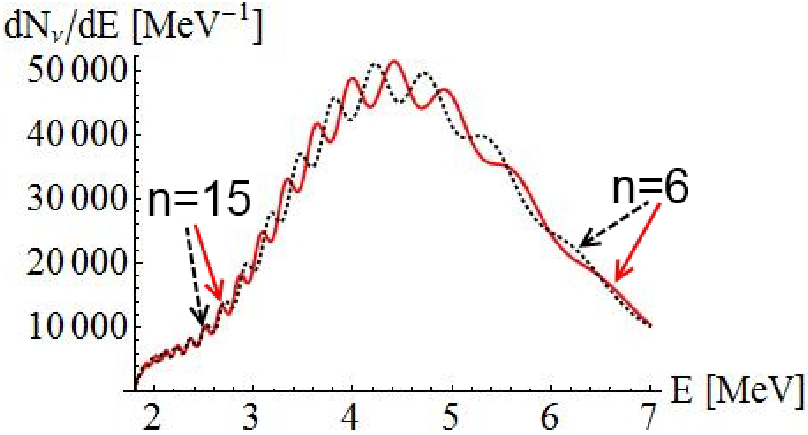
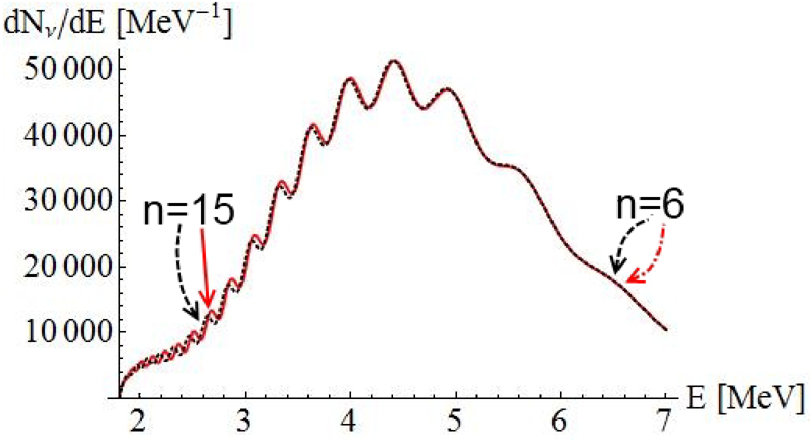


Figure: Theoretical Neutrino Spectrum from 17.4 GW of reactors observed in 24 years at a 5 kton target which is 10% hydrogen, including 3 flavor oscillation, for the normal (black dotted curve) and inverted (red curve) hierarchies as seen at 40 km, fixing $|\Delta M^2_{32}|$.

Now Add a 4% Relative Shift to $|\Delta M^2_{32}|$.



As above, but fixing $\Delta M^2_{\text{eff}} := \cos^2(\theta_{12})|\Delta M^2_{31}| + \sin^2(\theta_{12})|\Delta M^2_{32}|$ in both hierarchies.

The first 10 peaks alone cannot be used to determine the hierarchy, but the next 5 can.

Hierarchy from Peak Positions

Ref: Ciuffoli et al., JHEP 1303 (2013) 016 arXiv:1208.1991

The energy E_n of the nth peak satisfies

$$\frac{L}{E_n} = \frac{4\pi\hbar}{\Delta M^2_{(n)} c^3} \tag{1}$$

where we define the effective mass measured by the nth peak

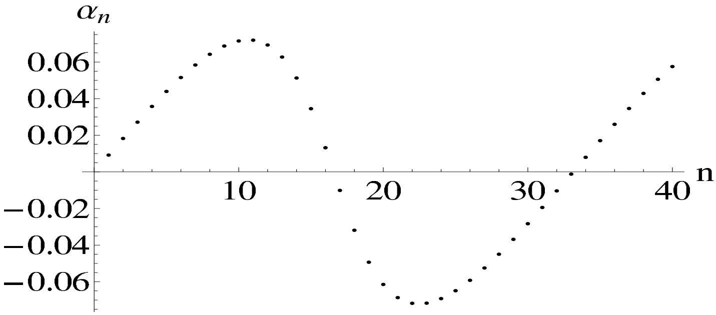
$$\Delta M^2_{(n)} = \frac{|\Delta M^2_{31}|}{1 \pm \alpha_n/n} \tag{2}$$

The $+$ (-) corresponds to the normal (inverted) hierarchy.

Strategy: You calculate α_n and measure *at least two* E_n.
Then use these Eqs. (1,2) to determine $|\Delta M^2_{31}|$ and the hierarchy.

The Function α_n

Ref: Ciuffoli et al., JHEP 1303 (2013) 016 arXiv:1208.1991



At $n << 10$, α_n/n is about 0.01, whereas $\alpha_{16} \sim 0$.

Therefore, fixing $\Delta M^2_{\mathrm{eff}}$ using a high energy peak, the energy E_{16} of the 16th peak will be 2% higher in the case of the normal hierarchy than in that of the inverted hierarchy.

Summary of how to determine the hierarchy

The mid and high energy part of the spectrum determines

$$\Delta M^2_{\rm eff} = \cos^2(\theta_{12})|\Delta M^2_{31}| + \sin^2(\theta_{12})|\Delta M^2_{32}|$$

The low energy part determines other combinations, for example the 16th peak determines $|\Delta M^2_{31}|$

Subtracting these:

$$\Delta M^2_{\rm eff} - |\Delta M^2_{31}| = \sin^2(\theta_{12})(|\Delta M^2_{32}| - |\Delta M^2_{31}|)$$

which is positive (negative) if the hierarchy is inverted (normal).

Note that the difference between the hierarchies is $2\sin^2(\theta_{12})\Delta M^2_{21}$ which is only 2% of $|\Delta M^2_{31}|$ - the energy must be determined very precisely.

Conclusion: The high and low energy peaks are both necessary

Resolution requirements

In particular the resolution should be good enough to be able to see oscillations beyond the 14th peak - $3\%/\sqrt{E/\mathrm{MeV}}$

This requires a 2x reduction of the state of the art.

The resolution is limited by statistical fluctuations in the number of photoelectrons, so 4x more photoelectrons are necessary.

Such a big increase requires a need new and better scintillator mix, very efficient PMTs and either a transparent scintillator (no Gd) or several small detectors

Due to optical quenching, less photoelectrons are seen from events further from the PMTs.

This causes a mild degeneracy between the event energy and its location, which is broken somewhat by the distribution of photons among the PMTs.

The degeneracy worsens the resolution, and so requires yet more photoelectrons.

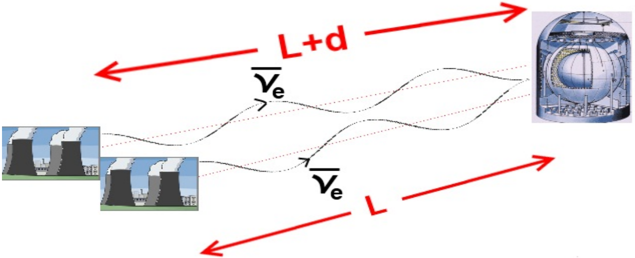
Interference

Ref: Ciuffoli et al., JHEP 1303 (2013) 016 arXiv:1208.1991

Reactors within a complex are often separated by of order 1 km.

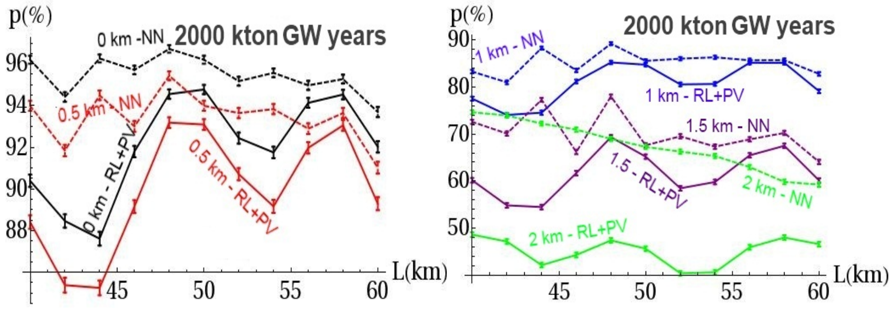
For example the reactors within the Hanbit complex lie along a line which is 1.3 km long.

This means that neutrinos from different reactors travel different distances, and for small energies (2.5 MeV) neutrinos from 1 reactor will be at their 1-3 maximum while neutrinos from another are at their minimum, erasing the 1-3 oscillation signal



Interference as a Function of Baseline Difference

Ref: Ciuffoli et al., arXiv:1302.0624

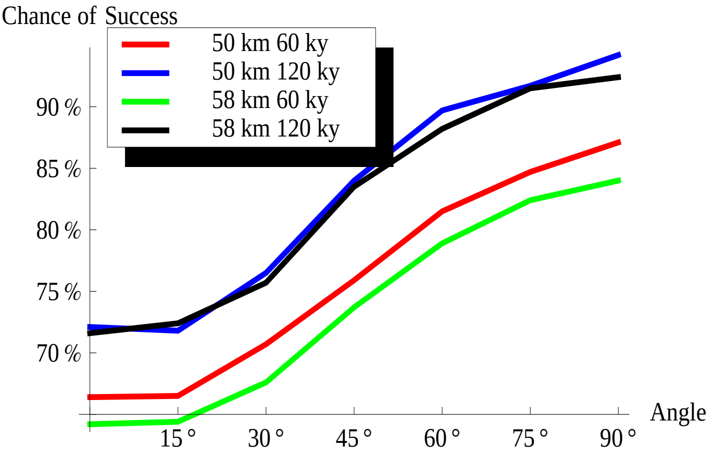


Here are the probabilities of successfully determining the hierarchy for experiments using two neutrino sources at different distances.

It is clear that if the distances differ by more than about 500 meters, the confidence in the hierarchy diminishes substantially.

Interference and Angle with respect to Reactor Complex

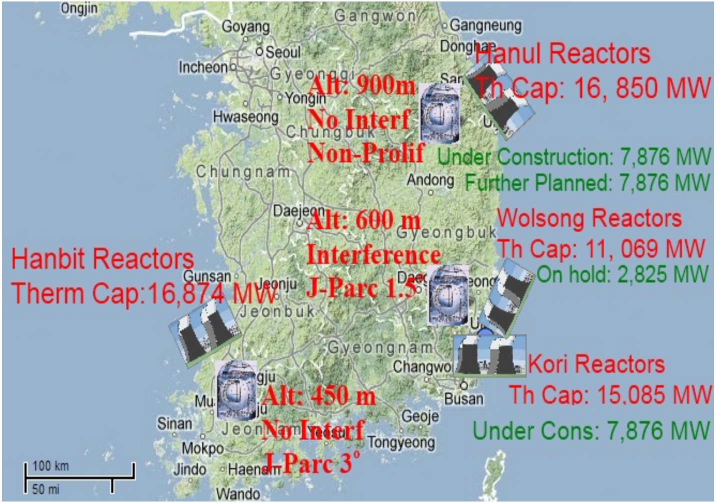
Ref: Ciuffoli et al., JHEP 1212 (2012) 004 arXiv:1209.2227



Interference and Sites for Daya Bay II

Ref: Ciuffoli et al., JHEP 1212 (2012) 004 arXiv:1209.2227

Sites for RENO 50



χ^2 Analysis - Wilks' Theorem

Ref: Wilks, Ann. Math. Stat. 9, No. 1 (1938), 60-62

To test a null hypothesis that a continuous variable $x = x_0$:

(1) Measure the quantities y_i and calculate the statistic

$$\chi^2(x) = \sum_i \frac{\left(y_i^{(\mathrm{measured})} - y_i^{(\mathrm{theoretical})}(x)\right)^2}{\sigma_i^2}$$

(2) Define $\overline{x}$ to be the value of x that minimizes χ^2.

(3) Define (note that this is always positive by def of $\overline{x}$):

$$\Delta\chi^2 = \chi^2(x_0) - \chi^2(\overline{x})$$

Wilks' Theorem: $\Delta\chi^2$ follows a 1 DOF χ^2 distribution

Conclusion:

The hypothesis $x = x_0$ is excluded with confidence $\sqrt{\Delta\chi^2}\ \sigma$

Wilks' Theorem Does Not Apply to the Hierarchy

Ref: Qian et al., Phys.Rev. D86 (2012) 113011

The neutrino mass hierarchy is *not* a continuous variable, it is a discrete variable.

In the case of the hierarchy one instead defines the statistic

$$\Delta\chi^2 = \chi^2_{(\mathrm{inv})} - \chi^2_{(\mathrm{nor})}$$

where $\chi^2_{(\mathrm{inv})}$ and $\chi^2_{(\mathrm{nor})}$ are the χ^2 statistics obtained by fitting with respect to each hierarchy, with nuisance parameters chosen separately to minimize each χ^2.

This is not the quantity described in Wilks' theorem, it is not even necessarily positive.

Therefore $\Delta\chi^2$ does not satisfy a χ^2 distribution.

χ^2 Analysis of the Hierarchy

Ref: Ciuffoli et al., arXiv:1305.5150

Under quite general conditions $\Delta\chi^2$ follows a Gaussian distribution centered at $\overline{\Delta\chi^2}$ with $\sigma = 2\sqrt{\overline{\Delta\chi^2}}$.

The mean and median probability of correctly determining the hierarchy are:

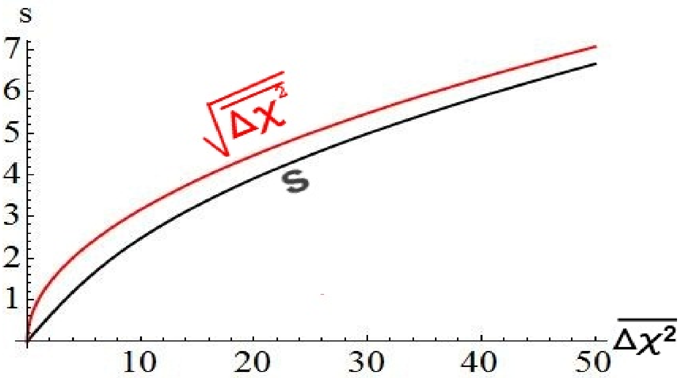
$$\frac{1}{2}\left(1+\mathrm{erf}\left(\sqrt{\frac{\overline{\Delta\chi^2}}{8}}\right)\right) \quad \text{and} \quad \frac{1}{1+e^{-\overline{\Delta\chi^2}/2}}$$

and the number of σ's of confidence of the median experiment is

$$s = \sqrt{2}\,\mathrm{erf}^{-1}\left(\frac{1-e^{-\overline{\Delta\chi^2}/2}}{1+e^{-\overline{\Delta\chi^2}/2}}\right)$$

Correct vs Incorrect χ^2 Analysis

Ref: Ciuffoli et al., arXiv:1305.5150



A naive application of Wilks' theorem (red) overestimates the confidence in the median experiment (black) by about 0.5σ.

Detector's Unknown Nonlinear Energy Response

Ref: Parke et al, Nucl.Phys.Proc.Suppl. 188 (2009) 115-117

Recall that the hierarchy signal is a 2% relative shift in the peak energies.

This requires that the statistical and the systematic errors in the energy be less than half of the state of the art.

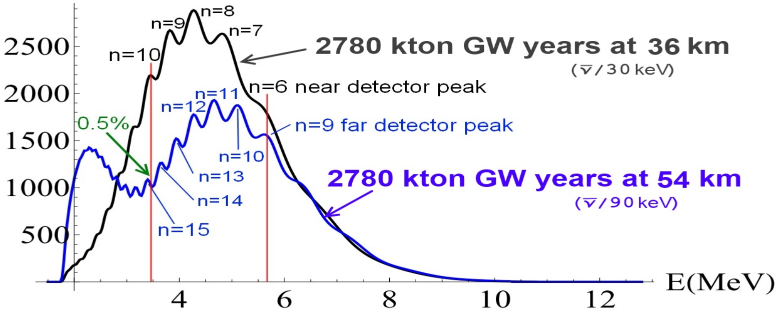
The statistical errors determine the resolution of the detector, discussed above.

The systematic error is the unknown part of the detector energy response, only the nonlinear part affects the relative energy differences and so the hierarchy.

The main challenge to the hierarchy determination at a reactor experiment is the unknown nonlinear energy response.

Ref: Ciuffoli et al., arXiv:1211.6818

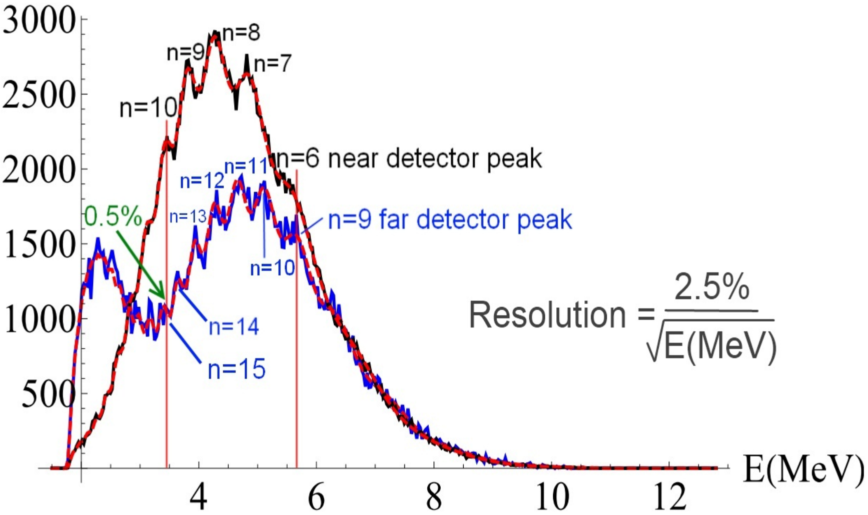
This problem can be circumvented in principle if one uses two identical detectors at distinct baselines: use only *relative* energy measurements - independent of the correlated systematic error



Here the 10th peak at 36 km is at a *higher* energy than the 15th peak at 54 km, so the hierarchy is inverted.

The Two Detector Proposal - Simulation

Ref: Ciuffoli et al., arXiv:1211.6818

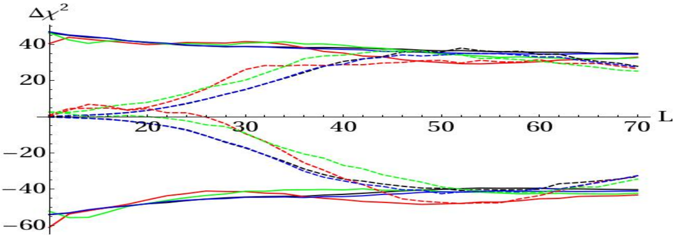


χ^2 Analysis of Nonlinearity with 1 or 2 Detectors

Work in Progress with Ciuffoli, Wang, Yang, Zhang and Zhong

$\Delta\chi^2$ for 1 detector (dashed) vs 1 detector at 55 km and 1 at an arbitrary baseline (solid), each with 8640 kton GW years total

A perfect energy response, a linear shift, quadratic model and Daya Bay best fit of nonlinearity to generate the spectra and a E+constant+$1/E$ fit is used to minimize each χ^2.



Two detector experiments outperform one detector experiments with the same total target mass and distinct baselines

Combining Reactor + Accelerator Disappearance Channels

Ref: Nunokawa et al., Phys. Rev. D 72, 013009 (2005)

The ν_μ disappearance channel at long baseline oscillation experiments determines the atmospheric mass difference

$$\Delta M^2_{\mathrm{atm}} = |\Delta M^2_{31}| \mp (\cos^2(\theta_{12}) - \cos(\delta)\sin(\theta_{13})\sin(2\theta_{12})\tan(\theta_{23}))\Delta M^2_{21}$$

The - (+) sign applies to the normal (inverted) hierarchy, in which case it is lower (higher) than the reactor neutrino mass differences.

Thus to determine the hierarchy it suffices to compare the atmospheric and reactor mass effective mass differences.

Quantitative Comparison

Minakata et al., Phys.Rev. D74 (2006) 053008, Ciuffoli et al. arXiv:1302.0624

For example, at medium and high energies recall that a medium baseline reactor experiment determines

$$\Delta M^2_{\mathrm{eff}} = \cos^2(\theta_{12})|\Delta M^2_{31}| + \sin^2(\theta_{12})|\Delta M^2_{32}|$$

while at low energies, the 16th peak determines $|\Delta M^2_{31}|$.

Comparing these with the atmospheric difference

$$\Delta M^2_{\mathrm{eff}} - \Delta M^2_{\mathrm{atm}} = \pm(2\cos(2\theta_{12}) - \cos(\delta)\sin(\theta_{13})\sin(2\theta_{12})\tan(\theta_{23}))\Delta M^2_{21}$$
$$|\Delta M^2_{31}| - \Delta M^2_{\mathrm{atm}} = \pm(\cos^2(\theta_{12}) - \cos(\delta)\sin(\theta_{13})\sin(2\theta_{12})\tan(\theta_{23}))\Delta M^2_{21}$$

The $\cos(\delta)$ term is always subdominant.

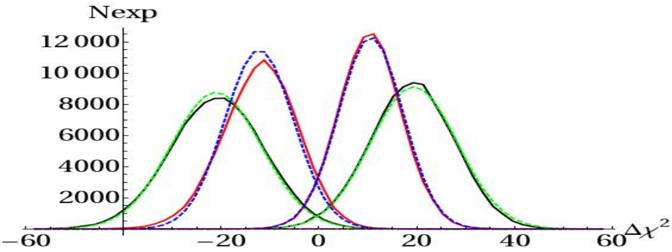
The signs of these differences provide two additional indicators of the hierarchy.

The greater $\cos(\delta)$, the smaller the mass difference and so the weaker the hierarchy signal.

Reactor and Accelerator when $\cos(\delta) = 0$

Ref: Ciuffoli et al., arXiv:1305.5150

Combining MINOS' 4% (upgraded NOνA's 1%) determination of $\Delta M^2_{\mathrm{atm}}$ with 6 years of Daya Bay II, out of 50,000 simulations per hierarchy we obtained the purple (green) distribution of $\Delta\chi^2$

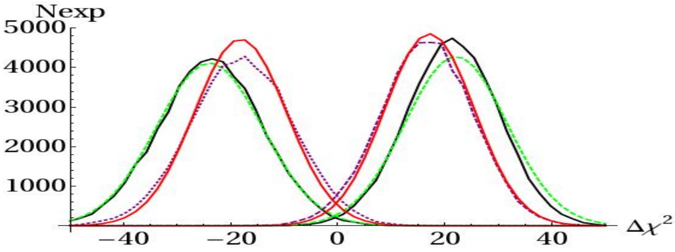


$\overline{\Delta\chi^2} \sim 11$ (20) for Daya Bay II with MINOS (NOνA) yielding 2.6σ (3.9σ) of confidence at the median experiment

Reactor and Accelerator when $\cos(\delta) = \pm 1$

Ref: Ciuffoli et al., arXiv:1305.5150

NOνA with 6 years of Daya Bay II using $\delta = 0$ and $\delta = \pi$



At $\delta = 0$ (π) we find $\overline{\Delta\chi^2} = 17$ (22) yielding 3.5σ (4.2σ) of confidence.

Towards $\cos(\delta)$

Ref: Ciuffoli et al., arXiv:1302.0624

NOνA and T2K have some sensitivity to sin(δ) as it determines the difference between ν_e and $\overline{\nu_e}$ appearance in the ν and $\overline{\nu}$ modes.

However due to a severe degeneracy with θ_{13} and θ_{23}, NOνA and T2K cannot distinguish δ from $\pi - \delta$.

In principle, the differences between the reactor and atmospheric effective masses depend upon $\cos(\delta)$ and so can break this degeneracy: For $\delta = 0$ (π)

$$\frac{\Delta M^2_{\mathrm{atm}} - \Delta M^2_{\mathrm{eff}}}{\Delta M^2_{\mathrm{eff}}} \sim 0.8\% \ (1.7\%)$$

To determine the hierarchy, $\Delta M^2_{\mathrm{eff}}$ and $|\Delta M^2_{31}|$ must both be measured to within 0.5%. As $\Delta M^2_{\mathrm{eff}}$ is easier to determine, its measurement will be better. The limiting factor will then be the measurement of $\Delta M^2_{\mathrm{atm}}$ at long baseline accelerator experiments.

Conclusions

The medium and high energy part of the spectrum determines $\Delta M^2_{\mathrm{eff}}$ and the low energy part $|\Delta M^2_{31}|$, the hierarchy is determined by comparing them

To avoid interference, the reactors must all be within 500 m of the same distance to the detector, which can be achieved if the detector is orthogonal to a linear reactor array

The median confidence of a hierarchy determination at *any* experiment is not $\sqrt{\Delta\chi^2}$, but is about one half σ less

The unknown energy response is a big problem, but can be addressed using identical detectors at distinct baselines

An accelerator experiment alone cannot distinguish δ from $\pi - \delta$, but perhaps in combination with a reactor experiment it can